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A note on strain localization for a class of non-associative plasticity rules*

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Summary: Explicit solutions for the formation of discontinuity bands are obtained, for a class of non-associative flow rules. Specialization to particular yield functions for pressure sensitive, dilatant or compactive materials is given.

Bemerkungen zur Lokalisierung der Verformungen für eine Klasse von nicht assoziierten plastischen Fließgesetzen

Übersicht: Es werden explizite Lösungen für die Bildung von Unstetigkeitsflächen bei einer Klasse von nicht assoziierten plastischen Fließgesetzen hergeleitet. Diese werden insbesondere für einige spezielle Fließgesetze diskutiert, die zur Beschreibung von druckempfindlichen, dilatierenden oder kontrahierenden Materialien geeignet sind.

1 Introduction

The occurrence of a particular type of non-uniqueness in the incremental elastoplastic response, in form of a strain rate discontinuity across a narrow planar band, was discussed by Rice and Rudnicki [1–3], and by Vardoulakis [4] in order to model strain localization in metals and rock-like materials.

The loading threshold corresponding to strain localization is attained when the acoustic tensor $\mathbf{nDn}$ becomes singular, i.e. for at least one direction:

$$\det(\mathbf{nDn}) = 0. \tag{1.1}$$

Here $\mathbf{D}$ is the constitutive fourth order tensor of incremental stiffness of the material and $\mathbf{n}$ is a versor orthogonal to the discontinuity band. The condition (1.1) corresponds to the vanishing of the speed of acceleration waves [5–8].

In the preceding works [9, 10] it is shown that, under the hypothesis of small strains and small rotations, it is possible to obtain an explicit expression for the critical hardening modulus, corresponding to the localization of deformations, for rate independent associative and non-associative elastoplasticity. The relation of such a modulus to the loss of positiveness of the second order work [11–14] was also discussed for plane stress and plane strain. In the case of plane strain and plane stress associative elastoplasticity it was shown [9] that the second order work necessarily vanishes at the formation of a shear band. However, this may not be the case when a localization into a splitting discontinuity is attained.

In this paper, using the general method presented in [10], the conditions for strain localization are obtained for elastoplastic materials with a particular type of non-associative flow rule. The flow rule considered describes the behavior of pressure sensitive, dilatant or compactant materials [1, 15, 16]. Moreover, coupled elastoplastic deformation of highly porous materials with elastic

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bulk modulus increasing due to void collapse can be described by such a flow rule [17]. Explicit expressions for the critical hardening modulus (corresponding to the localization) and for the inclination of the band are finally given for three particular yield functions. Conditions for different forms of discontinuity bands, i.e. corresponding to shear or splitting modes, are discussed.

2 Basic relationships

The behaviour of the isotropic, homogeneous elastoplastic material considered here is characterized by

– a smooth yield surface, function of the stress $\boldsymbol{\sigma}$ and of the hardening parameter k:

$$f(\boldsymbol{\sigma}, k) = 0, \qquad \dot{k} = \dot{k}(\dot{\boldsymbol{\epsilon}}^p, \dot{\boldsymbol{\sigma}}); \tag{2.1}$$

– a flow rule of the plastic strain rate tensor $\dot{\boldsymbol{\epsilon}}^p = \dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^e$

$$\dot{\boldsymbol{\epsilon}}^p = \dot{\Lambda}\mathbf{P}; \tag{2.2}$$

– the stress-strain law

$$\dot{\boldsymbol{\sigma}} = \mathbf{E} : \dot{\boldsymbol{\epsilon}} - \dot{\Lambda}\mathbf{E} : \mathbf{P}, \tag{2.3}$$

subjected to the conditions

$$\dot{\Lambda} \geqq 0, \qquad \dot{f} \leqq 0, \qquad \dot{f}\dot{\Lambda} = 0; \tag{2.4}$$

– the plastic hardening modulus H

$$H = -\frac{\partial f}{\partial \boldsymbol{\epsilon}^p} : \mathbf{P}; \tag{2.5}$$

– the elastoplastic stiffness tensor $\mathbf{D}^{ep}$, obtained from (2.1)–(2.5)

$$\dot{\boldsymbol{\sigma}} = \mathbf{D}^{ep} : \dot{\boldsymbol{\epsilon}}, \qquad \mathbf{D}^{ep} = \mathbf{E} - \frac{(\mathbf{P} : \mathbf{E}) \otimes (\mathbf{Q} : \mathbf{E})}{H + \mathbf{P} : \mathbf{E} : \mathbf{Q}}. \tag{2.6}$$

In (2.1)–(2.6) a dot denotes derivative with respect to time. Moreover, $\mathbf{P}$ is a symmetric second order tensor determining the mode of the plastic flow, different (non-associative plasticity) from the yield surface gradient $\mathbf{Q}$, $\dot{\Lambda}$ is referred to as the plastic multiplier, $\dot{\boldsymbol{\epsilon}}^e$ represents the elastic strain rate, and $\mathbf{E}$ indicates the isotropic fourth order elastic tensor. Due to the non-associativity of the plastic strain rate to the yield surface ($\mathbf{P} \neq \mathbf{Q}$), the tensor $\mathbf{D}^{ep}$, given by (2.6), does not possess the major symmetries.

In what follows, the particular form

$$\mathbf{P} = \mathbf{Q} + \xi\boldsymbol{\delta} \tag{2.7}$$

of the tensor $\mathbf{P}$ will be considered where ξ is any scalar function and $\boldsymbol{\delta}$ is the Kronecker delta. In the case $\xi = 0$ the associative flow rule is recovered. The condition (2.7) restricts the non-associativity of the flow rule to the volumetric component of the plastic strain rate. This assumption is valid for many media of engineering importance, particularly for granular materials, whose plastic volumetric changes are ill estimated when an associative flow rule is assumed. Experimental evidence [18, 19] shows at the same time that the deviatoric plastic strain rate follows the normality rule.

The threshold in terms of hardening modulus, corresponding to the bifurcation in the form of a localization into a planar band was obtained from (1.1) by Rice [2], Rice and Rudnicki [3]. The localization takes place when the hardening modulus reaches the critical values H_{cr}^l, obtained as the solution of the constrained maximization problem

$$H_{cr}^l = 2G \max_{\mathbf{n}} \left[2\mathbf{nPQn} - (\mathbf{nPn})(\mathbf{nQn}) - \mathbf{P} : \mathbf{Q} - \frac{\nu}{1-\nu}(\mathbf{nPn} - \operatorname{tr}\mathbf{P})(\mathbf{nQn} - \operatorname{tr}\mathbf{Q}) \right], \tag{2.8}$$

subjected to $|\mathbf{n}| = 1$, where G is the elastic shear modulus, ν is Poisson's ratio and tr indicates the trace of a tensor.

The velocity discontinuity vector g which defines the strain rate jump across the band for the **n** which maximizes (2.8) is

$$\mathbf{g} = 2\mathbf{nP} - \frac{1}{1-\nu}(\mathbf{nPn})\,\mathbf{n} + \frac{\nu}{1-\nu}(\mathrm{tr}\,\mathbf{P})\,\mathbf{n}. \tag{2.9}$$

A vector g normal to the versor **n** describes a localization mode corresponding to a simple shear deformation rate, whereas when g is parallel to **n** a splitting mode discontinuity occurs. The term shear band will be used to denote all the cases where g and **n** are not parallel. On the other band, the term splitting mode will denote co-axiality of g and **n**, i.e. simple extension or compression strain rates into the band. It may be observed that the calculation of the critical hardening modulus in (2.8) is coupled with the determination of the vector **n**. Ad hoc numerical procedures were therefore developed [20] to arrive at the value of the critical hardening modulus.

It is worth noting that, if the tensor **D** in (1.1) is identified with the tensor $\mathbf{D}^{ep}$ of (2.6), the solution (2.8) represents the threshold to localization of the comparison solid corresponding to the plastic loading branch. This is shown [14] to provide an upper bound to strain localization. A lower bound is obtained from (1.1) by identifing **D** with the symmetrized incremental stiffness tensor of the comparison solid introduced by Raniecki and Bruhns [14]. Under the hypothesis of single, smooth yield surface and plastic potential it was shown [3] that the comparison solid corresponding to the plastic loading branch sets the lower limit to localization. Thus, in what follows, only the localization of the comparison solid in loading will be considered.

First, however, it will be shown that the critical modulus and the vector **n** may each be obtained in an explicit de-coupled form if an appropriate reference system is employed.

3 Localization criterion

In this section an explicit form of the expression for the critical hardening modulus will be derived for localization into a planar band, in the case of general unconstrained kinematics. The constrained maximization problem (2.8) may be solved explicitely leading to uncoupled expressions for the critical modulus and the discontinuity band direction if a suitable system of reference is chosen. To that end the problem (2.8) is rewritten as the unconstrained maximization of the Lagrangean function

$$L(\mathbf{n}, \omega) = 2G\left[2\mathbf{nPQn} - (\mathbf{nPn})\,(\mathbf{nQn}) - \mathbf{P}:\mathbf{Q} - \frac{\nu}{1-\nu}(\mathbf{nPn} - \mathrm{tr}\,\mathbf{P})(\mathbf{nQn} - \mathrm{tr}\,\mathbf{Q})\right] - \omega(\mathbf{nn} - 1) \tag{3.1}$$

where ω is a Langrangean multiplier. By specializing the tensor **P** in the form (2.7) and choosing the principal axes of stress as the reference system, the maximization of function (3.1) becomes equivalent to the solution of the system of equations

$$\begin{aligned} &2n_i(Q_i + \xi)\,Q_i - \frac{1}{1-\nu}\left[(n_i^2 Q_i + n_j^2 Q_j + n_k^2 Q_k)\,(2Q_i n_i + \xi n_i) + \xi Q_i n_i\right] \\ &+ \frac{\nu}{1-\nu}\left[(Q_i + Q_j + Q_k)\,(\xi + 2Q_i) + 3\xi Q_i\right] n_i = \omega n_i/(2G), \\ &n_i^2 + n_j^2 + n_k^2 = 1 \end{aligned} \tag{3.2}$$

where the indices i, j, k (no summation) refer to the components in the reference system of principal stresses and are to be permuted in the range I, II and III. The system of four equations (3.2) has four independent variables which are the three components of versor the **n** and the Langrangean multiplier ω. The values of the components of **n** which are solutions of the system (3.2) yield the extrema moduli H^l of (3.1).

Let us examine the following possible cases of the orientation of the discontinuity band in terms of its normal versor **n** in the reference system of principal stress directions:

i. None of the components of **n** is null.
ii. One component of **n** is null,
iii. Two components of **n** are null.

i. If all components of the versor **n** are different from zero, the system (3.2) cannot have a unique solution. This conclusion is obtained examining the determinant of the matrix of coefficients which is always zero. Consequently, the following three cases are possible:

1. The tensor **Q** does not have any symmetry:

$$Q_{\mathrm{I}} \neq Q_{\mathrm{II}}, \qquad Q_{\mathrm{II}} \neq Q_{\mathrm{III}}, \quad Q_{\mathrm{I}} \neq Q_{\mathrm{III}}. \tag{3.3}$$

In this case the system (3.2) does not possess any solution. The extrema of (3.1) are to be sought in the cases when at least one component of the versor **n** is zero (cases ii. and iii.).

2. The tensor **Q** is symmetric with respect to the axis k:

$$Q_i = Q_j, \qquad Q_j \neq Q_k. \tag{3.4}$$

The system (3.2) admits now ∞^1 solutions. The inclination of the band with respect to the axis k is uniquely determined from (3.2). On the other hand, all the possible combinations of components n_i and n_j yield the same extremum value of (3.1). The component n_k and the corresponding value of the hardening modulus may be obtained directly, without loss of generality because of the symmetry, by referring to the case in which one of the components n_i or n_j is assumed to be zero (case ii.) If this value of the hardening modulus is critical, i.e. if it is the maximum over all the extrema, then the number of the possible shear bands becomes infinite, corresponding, as indicated in Fig. 1, to an inclination which is indeterminate with respect to the axes i and j.

3. The tensor **Q** is symmetric with respect to all the principal axes of stress:

$$Q_{\mathrm{I}} = Q_{\mathrm{II}} = Q_{\mathrm{III}}. \tag{3.5}$$

Now the system (3.2) admits ∞^2 solutions. The number of possible discontinuity bands is infinite and the inclination is indeterminate. The critical hardening modulus is obtained, without loss of generality, again assuming one of the components of the versor **n** as equal to unity and the two other as equal to zero (case iii.). It is worth noting that in this case the localization mode is a splitting discontinuity.

ii. When one of the components of the versor **n** is zero, the normal to the discontinuity band is orthogonal to one of the principal stress directions. Suppose $n_k = 0$. Two cases are possible in which the tensor **Q** is or is not symmetric with respect to the axis k.

1. The tensor **Q** is not symmetric with respect to the axis k:

$$Q_i \neq Q_j. \tag{3.6}$$

Then (3.2) yields

$$n_i^2 = \frac{Q_i + \nu Q_k}{Q_i - Q_j} + \frac{\xi(1 + \nu)}{2(Q_i - Q_j)}, \qquad n_j^2 = 1 - n_i^2, \qquad n_k = 0. \tag{3.7}$$

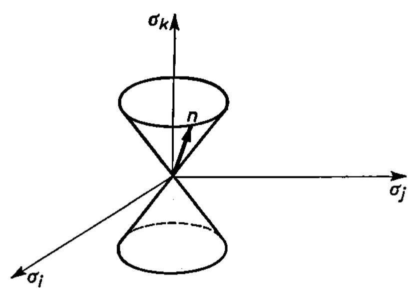

Fig. 1. $Q_i = Q_j$ shear band localization mode: Possible directions of versor **n**

Note that, in order to be an admissible solution, the value of n_i^2 must belong to the interval [0, 1]. This imposes restrictions on the components of **Q** in (3.7). When the above condition is not satisfied, the extremum of the modulus (3.1) occurs when two of the components of the versor **n** are zero (case iii.). When admissible, (3.7) represents the only solution corresponding a inclined shear band with respect to the principal stress axes.

Substituting (3.7) into (3.1), the following expression for the corresponding hardening modulus is obtained:

$$H^l = (1 + \nu)\frac{G}{2}\left(\zeta^2 \frac{1 + \nu}{1 - \nu} - 4\xi Q_k - 4Q_k^2\right). \tag{3.8}$$

From (3.8) it may be seen that, in the case of associative plasticity ($\xi = 0$), the critical hardening modulus cannot be positive. For non-associative plasticity ($\xi \neq 0$) the critical hardening modulus may be positive, as in the case of $Q_k = 0$.

The discontinuity vector **g**, corresponding to the band inclination defined through (3.7), is obtained from (2.9) as

$$\begin{aligned} g_i &= [Q_i - Q_j + \xi(1 + \nu)/(2 - 2\nu)]\, n_i, \\ g_j &= [Q_j - Q_i + \xi(1 + \nu)/(2 - 2\nu)]\, n_j, \qquad g_k = 0 \end{aligned} \tag{3.9}$$

where indices are not summed.

2. The tensor **Q** is symmetric with respect to the axis k:

$$Q_i = Q_j. \tag{3.10}$$

In this case the system (3.2) admits ∞^1 solutions. All the inclinations in respect to the axes i and j yield the same value for the extremum of (3.1). The corresponding hardening modulus is obtained, without loss of generality, by assuming one of the components of the versor **n** to be zero in the i–j plane (case *iii.*). If this value of the hardening modulus is critical, i.e. it is the maximum over all the extrema, then an infinite number of shear bands of indeterminate inclination is possible (Fig. 2). The localization mode is a splitting discontinuity.

iii. If two of the components of the versor **n** are zero, i.e. the normal to the discontinuity band is orthogonal to two of the principal axes of stress, e.g.

$$n_i = 1, \qquad n_j = n_k = 0, \tag{3.11}$$

the corresponding hardening modulus becomes

$$H^l = -\frac{2G}{1 - \nu}\,[(Q_j + \nu Q_k)^2 + \xi(1 + \nu)\,(Q_k + Q_j)] - 2G(1 + \nu)\,Q_k^2. \tag{3.12}$$

Note that, in the associative case ($\xi = 0$), the hardening modulus (3.12) is negative and always smaller than the modulus (3.8).

The vector **g** is obtained from (3.11) and (2.9) as

$$g_i = Q_i + \nu(Q_j + Q_k)/(1 - \nu) + \xi(1 + \nu)/(1 - \nu), \qquad g_j = g_k = 0 \tag{3.13}$$

where indices are not summed. From the components (3.13) of **g** it may be noted that the localization mode is necessarily a splitting discontinuity.

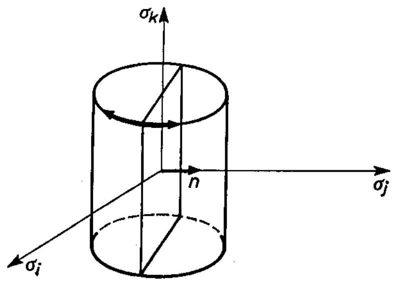

Fig. 2. $Q_i = Q_j$ splitting discontinuity mode: Possible directions of versor **n**

In a generic case, without a priori determined direction of $\mathbf{n}$, the critical hardening modulus is to be found as the maximum over all the possible extrema. Thus, in order to calculate the critical hardening modulus, all the extrema are to be evaluated in the way indicated in the subsections i., ii. and iii. where the indices are to be permuted in the range I to III. It is to be observed that in all the cases this reduces to the calculation of inclinations through (3.7) and of the moduli (3.8) and (3.12).

It is important to note that, excluding the special case of infinite solutions, the normal to the band is always orthogonal to a principal stress direction.

All the above results reduce to previous findings by Rudnicki and Rice [1] for the case of Huber/von Mises and Drucker/Prager yield functions. For an arbitrary yield surface and non-associative flow rule a decoupled solution can be attained employing the maximization procedure for the problem (3.1), see [10] In the next section the various possible localization modes are discussed.

4 On localization modes

From the above section it may be concluded that the splitting mode localization always occurs in the following cases:

- The tensor $\mathbf{Q}$ is symmetric with respect to all principal stress directions;
- the modulus (3.12) is the one that maximizes (3.1).

On the other hand, the shear bands may form when the maximum of expression (3.1) corresponds to modulus (3.8) and the corresponding band inclination (3.7) is admissible.

The condition under which deformation discontinuity mode is a simple shear strain rate band is defined by

$$\mathbf{g}\mathbf{n} = 0. \tag{4.1}$$

Using (3.7) and (3.9), the condition (4.1) yields

$$Q_i + Q_j + 2\nu Q_k + \xi(1+\nu)(3-2\nu)/(1-\nu) = 0. \tag{4.2}$$

Note that, for associative plasticity, the modulus (3.8) is always smaller than the modulus (3.12), but the inclination (3.7), corresponding to (3.8), may be non-admissible.

Let us consider the special associative case, including the Huber/von Mises model, when the tensor $\mathbf{Q}$ is expressed in the form

$$\mathbf{Q} = A\mathbf{s} \tag{4.3}$$

where A is any scalar function and $\mathbf{s}$ is the deviatoric stress. In absence of kinematical or statical constraints, as plane strain or plane stress conditions, the inclination (3.7) is always admissible. Consequently the critical hardening modulus, is

$$H_{cr}^l = -2G \max_{m=\mathrm{I,II,III}} [(1+\nu) A s_m^2]. \tag{4.4}$$

Let k be the value of index m that maximizes (4.4). The condition (4.2) becomes

$$s_k(1-2\nu) = 0. \tag{4.5}$$

If the condition (4.5) is satisfied for the value of the hardening modulus defined by (4.4), then localization of deformation is possible into a simple shear mode band. From (3.7), the band turns out to be inclined at 45° with respect to the principal stress axes i and j.

Excluding the special case when $\nu = 0.5$, the condition (4.3) implies that a simple shear discontinuity band may occur at a null value of the hardening modulus if the intermediate principal deviatoric stress reduces to zero. Thus, in a loading process in the presence of an associative flow rule (4.3), the first possibility of localization occurs at a null value of the plastic hardening modulus, when the intermediate principal component of stress deviator vanishes.

Let us now remember that, for associative elastoplasticity, the second order work vanishes, at a null value of the plastic hardening modulus [11–14]. Thus, in a loading process of a body with an associative flow rule of type (4.3), the possibility of a simple shear deformation band always arises at the vanishing of the second order work. In these conditions, since the positive second order work criterion represents a sufficient condition for uniqueness, any bifurcation is excluded before the possibility of formation of simple shear bands.

5 Specialization of the criteria

To complement the results of Rudnicki and Rice [1] for Drucker/Prager and Huber/von Mises yield functions, criteria for the critical hardening moduli and inclinations of the possible band will be derived now for three more advanced yield surfaces for pressure sensitive, dilatant or compactive materials.

– Roscoe and Burland [21] formulated the modified cam-clay yield condition

$$f = (\operatorname{tr} \boldsymbol{\sigma} - A)^2 + BJ_2 - C = 0 \tag{5.1}$$

where A, B and C are functions of volumetric plastic strain; and J_2 is the second invariant of the deviatoric stress, i.e.

$$J_2 = \mathbf{s} : \mathbf{s}/2 .$$

The yield surface gradient $\mathbf{Q}$ results to be

$$\mathbf{Q} = 2(\operatorname{tr} \boldsymbol{\sigma} - A)\, \boldsymbol{\delta} + B\mathbf{s} . \tag{5.2}$$

– Ottosen [22] stated the four parameter yield function

$$f = AJ_2/f_c^2 + \lambda \sqrt{J_2}/f_c + B \operatorname{tr} \boldsymbol{\sigma}/f_c - 1 = 0 \tag{5.3}$$

where the parameters A, B are taken as functions of a hardening parameter and f_c is the uniaxial compression resistance. Moreover, λ is a function of the stress invariant angle of similarity

$$\vartheta = \frac{1}{3} \cos^{-1} \left(\frac{3\sqrt{3}\, J_3}{2J_2^{3/2}} \right), \tag{5.4}$$

where $J_3 = \det \mathbf{s}$ is the third deviatoric stress invariant. Here, for the sake of simplicity, λ will be considered constant, i.e. a three-parameter model is adopted. Note that, when $A = 0$, the Drucker/Prager yield criterion is recovered. The tensor $\mathbf{Q}$ is now

$$\mathbf{Q} = B\boldsymbol{\delta}/f_c + \left[A/f_c^2 + \lambda/\left(2f_c\sqrt{J_2}\right) \right] \mathbf{s} . \tag{5.5}$$

– Bresler and Pister [23] presanted the three-parameter yield function

$$f = B \operatorname{tr} \boldsymbol{\sigma}/(3f_c) + \sqrt{2J_2/5}/f_c - C(\operatorname{tr} \boldsymbol{\sigma})^2/(9f_c^2) - A = 0 \tag{5.6}$$

where A, B and C depend on amount of hardening. Also in this case the Drucker/Prager model is obtained when $\mathbf{C} = 0$. The tensor $\mathbf{Q}$ is now

$$\mathbf{Q} = [B/(3f_c) - 2C \operatorname{tr} \boldsymbol{\sigma}/(9f_c^2)]\, \boldsymbol{\delta} + \mathbf{s}/\left(f_c\sqrt{10J_2}\right). \tag{5.7}$$

In all cases (5.2), (5.5) and (5.7) the tensor $\mathbf{Q}$ is expressible in the form

$$\mathbf{Q} = \alpha\boldsymbol{\delta} + \beta\mathbf{s} \tag{5.8}$$

where α and β are functions of plastic strain hardening parameters and possibly of the stress invariants $\operatorname{tr} \boldsymbol{\sigma}$ and J_2.

Let us exclude the cases of infinite solutions for the inclination of the band. In this way the generality is not lost because the critical hardening modulus is anyway given by (3.8) or (3.12). For the three models presented, the inclination of the band is given by (3.7) (if admissible) or (3.11). The critical hardening modulus is therefore the maximum between the two cases of an inclined shear band, (3.8), and of a splitting mode band orthogonal to a principal stress axis, (3.12). Making use of (4.8), (3.8) gives

$$H^l = (1 + \nu) \frac{G}{2} \max_{k=\text{I,II,III}} \left[\xi^2 \frac{1 + \nu}{1 - \nu} - 4(\alpha + \beta s_k) (\xi + \alpha + \beta s_k) \right] \tag{5.9}$$

corresponding to the inclination

$$n_i^2 = \frac{(\xi + 2\alpha)(1 + \nu) + 2(s_j + \nu s_k)}{2\beta(s_i - s_j)}, \quad n_k = 0, \quad n_j^2 = 1 - n_i^2. \tag{5.10}$$

When the inclination of the band is given by (3.11), the hardening modulus (3.12) becomes

$$H^l = \max_{j,k=\text{I,II,III}} \left\{ -\frac{2G}{1 - \nu} [(\alpha + \beta s_k)^2 + \xi(1 + \nu)(2\alpha + \beta s_k + \beta s_j) + 2\nu(\alpha + \beta s_k)(\alpha + \beta s_j) + (\alpha + \beta s_j)^2] \right\} \tag{5.11}$$

where the maximization refers to every permutation of indices j, k in the range I to III. The critical hardening modulus is the major one among the two given by (5.9) and (5.11), and the inclination of the band is given by the corresponding equations (5.10) or (3.11).

6 Conclusions

The critical hardening modulus for the localization and inclination of the discontinuity band has been obtained for a class of non-associative flow rules. The procedure provides simple de-coupled expressions equivalent to the condition of singularity of the acoustic tensor. The conditions for the occurrence of the localization modes corresponding to a shear or a splitting discontinuity are discussed. Reference is made to a simple associative elastoplastic model. Applications are finally performed with reference to three yield functions modelling pressure sensitive, dilatant or compactant, elastoplastic materials.

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References

1. Rudnicki, J. W.; Rice, J. R.: Conditions for the localization of deformations in pressure-sensitive dilatant materials. J. Mech. Phys. Sol. 23 (1975) 371—394
2. Rice, J. R.: The localization of plastic deformation. In: Koiter, W. T. (ed.) Theoretical and applied mechanics, pp. 207—220. Amsterdam: North-Holland 1976
3. Rice, J. R.; Rudnicki, J. W.: A note on some features of the theory of localization of deformation. Int. J. Sol. Struct. 16 (1980) 597—605
4. Vardoulakis, I.: Equilibrium theory of shear bands in plastic bodies. Mech. Res. Comm. 3 (1976) 209—214
5. Hadamard, J.: Leçons sur la propagation des ondes et les equations de l'hydrodynamique. Paris: Hermann 1903
6. Thomas, T. Y.: Plastic flows and fracture of solids. New York: Academic Press 1961
7. Hill, R.: Acceleration waves in solids. J. Mech. Phys. Sol. 10 (1962) 1—16
8. Mandel, J.: Conditions de stabilite et postulat de Drucker. In: Kravtchenko, J.; Sirieys, P. M. (eds.) Rheology and soil mechanics, pp. 58—68. Berlin: Springer 1966
9. Bigoni, D.; Hueckel, T.: On uniqueness and strain localization in plane strain and plane stress elastoplasticity. Mech. Res. Comm. 17 (1990) 17—23
10. Bigoni, D.; Hueckel, T.: Uniqueness versus localization in associative and non-associative elastoplasticity (submitted)
11. Mróz, Z.: Non-associated flow laws in plasticity. J. de Mècanique 2 (1963) 21—42
12. Hueckel, T.; Maier, G.: Incremental boundary value problems in the presence of coupling of elastic and plastic deformations: a rock mechanics oriented theory. Int. J. Sol. Struct. 13 (1977) 1—15
13. Maier, G.; Hueckel, T.: Non associated and coupled flow-rules of elastoplasticity for rock-like materials. Int. J. Rock Mech. Min. Sci. 16 (1979) 77—92

14. Raniecki, B.; Bruhns, O. T.: Bounds to bifurcation stresses in solids with non-associated plastic flow law at finite strain. J. Mech. Phys. Sol. 29 (1981) 153–172
15. Bigoni, D.; Laudiero, F.: The quasi-static finite cavity expansion in a non-standard elasto-plastic medium. Int. J. Mech. Sci. 31 (1990) 825–837
16. Prevost, J. H.: Localization of deformations in elastic-plastic solids. Int. J. Num. Anal. Meth. Geomech. 8 (1984), 187–196
17. Hueckel, T.: Coupling of elastic and plastic deformation of bulk solids. Meccanica 11 (1976) 227–235
18. Lade, P. V.; Duncan, J. M.: Cubical triaxial tests on cohesionless soil. Proc. ASCE: SM. J. Soil Mech. Found. Div. 99 (1973) 793–812
19. Lade, P. V.; Duncan, J. M.: Elastoplastic stress-strain theory for cohesionless soil. Proc. ASCE: GT. J. Geot. Eng. Div. 101 (1975) 1037–1053
20. Ortiz, M.; Leroy, Y.; Needelman, A.: A finite element method for localized failure analysis. Comp. Meths. Appl. Mech. Engrg. 61 (1987) 189–214
21. Roscoe, K.; Burland, J.: On generalized stress-strain behaviour of wet clay. In: Haymann, J.; Leckie, F. A. A. (eds.) Engineering plasticity. pp. 535–609 Cambridge University Press 1968
22. Ottosen, N. S.: A failure criterion for concrete. Proc. ASCE: EM. J. Eng. Mech. Div. 103 (1977) 527–535
23. Bresler, B.; Pister, K. S.: Strength of concrete under combined stresses. J. American Concrete Inst. 30 (1958) 321–345

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