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A MODEL FOR TEACHING ELASTIC FRAMES

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ABSTRACT

A demonstration model was designed, produced and successfully tested in the classroom and in public scientific demonstrations to help in understanding the way elastic frames deform. The model consists of a two-story, one-bay planar frame, which can be deformed by applying a concentrated force at different positions. This force is measured (through a load transducer) and transmitted to a computer, which calculates the structure's deformation (through linear elastic analysis) in real time and displays its deformed shape on a monitor. The user of the model can compare the real deformation to the calculated one and qualitatively assess the validity of linear elastic analysis.

Keywords: *Demonstration model, structural mechanics, elastic structure.*

1. INTRODUCTION

Frame structures are basic elements in machines and common in modern architecture, so that students of Mechanical and Civil Engineering are specifically taught to calculate and design them. Students are trained to 'visualize' the deformation of these structures and to associate such a deformation with the diagram of bending moments, since the deformation of a frame is mainly produced by the inflexion of beams and columns (in contrast to truss structures where the rods are axially deformed). This training is essential, for the reasons already clear to Cross and Morgan (1932) [1], who wrote: '*the ability of a designer of continuous structures is measured chiefly by his ability to visualize the deformation of the structure under load. If he cannot form a rough picture of these deformations when he begins the analysis he will probably analyse the structure in some very awkward and difficult way; if he cannot picture these deformations after he has made the analysis, he doesn't know what he is talking about*', a statement which remains fully valid. However, the visualization of the deformation of a structure with elements subject to bending is more complex than it may appear and for this reason the importance of teaching models to stimulate students' interest and simplify learning is well known. Such models have been developed and described by Pippard (1947) [2], Godden (1962) [3], Charlton (1966) [4], Hilson (1972) [5], and Harris and Sabnis (1999) [6].

In line with a renewed interest in teaching models for structures (Ji and Bell, 2008 [7]; Bigoni *et al.*, 2012 [8]) and in laboratory experiments for teaching mechanical concepts (Farver and Boughton, 2012 [9]; Wright *et al.*, 2013 [10]), the present article shows that teaching models for elastic frames, designed and realized following classical methodologies (Pippard, 1947) [2], can be enhanced through the use of modern digital technology. In this way, a demonstration tool is provided to show to both students of engineering and the untrained public how successful and predictive engineering design can be, as a fundamental step towards a simplification in teaching mechanics in the 21st century (Mascarenhas, 2010 [11]; Krause *et al.*, 2010 [12]; Packham, 2012 [13]) .

In particular, a two-storey, one-bay planar frame was designed and constructed. The frame can be loaded at four different fixed positions, using a compatible loading device on which a load cell is mounted. The load cell transmits the contact force to a computer and the deformation of the frame is calculated, using linear elastic methods in real time and displayed on a monitor. As a result, the user can immediately visually compare the deformed shape of the physical model with theoretical results from linear elastic calculations. The presented model has been used in classroom demonstration (for undergraduate students of Civil and Mechanical Engineering) and for public demonstrations (at the so-called 'researchers' night' and at other events). The beauty of the model is that it highlights the accuracy of engineering modelling. A movie with a demonstration of the teaching model is available at *http://ssmg.unitn.it*/.

2. THE TEACHING MODEL

The demonstration frame is a planar model of a two-story, one-bay configuration, fixed at its base, and having perfect column-beam connections, (Figure 1). The two beams and the two columns (of length 600 mm) were constructed using solid polycarbonate (white 2099 Makrolon UV from Bayer, elastic modulus 2350 MPa) elements, having a cross section of 4 mm (thickness) × 40 mm (width). The connections at the frame base and between the four elements were constructed by bolting twelve L-shaped and two flat aluminum profiles at the joints of the structure, (Figure 2). The resulting structure has the ability to display visible deformation when loaded by hand.

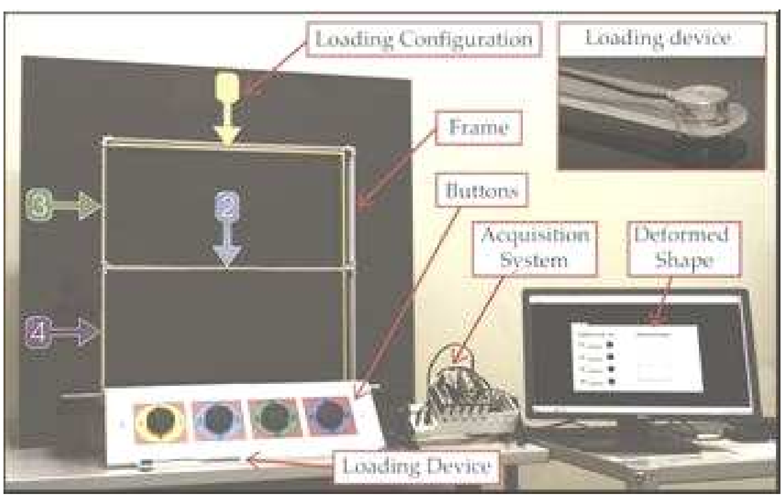

Figure 1. The two-storey, one-bay frame teaching model. After selecting of the loading mode using the color-coded buttons, the load is applied by the user at the points indicated by the arrows in the black background of the frame using the compatible loading device on which a load cell is mounted (see the inset). The load is acquired (through a NI CompactDAQ 9172 system by National Instrument©) and transmitted to a computer (not shown in the picture), which calculates the deformation of the frame in real time and displays the calculated image on the monitor (on the right).

Figure 2. Details of the beam column joints at the second (left) and first (centre) storey and of the clamp-constraint at the base (right) realized with L-shaped and flat aluminum profiles bolted to the solid polycarbonate flexible elements.

The load is applied by the user on the structure through a compatible loading device (inset of Fig. 1). This device is made of a stiff transparent link on which a miniaturized load cell (Leane CMM2-K100, R.C. 1000 N) is mounted; this cell provides the measure of the applied force.

Four different positions and related load directions were defined for the application of the load on the frame. These positions are labeled from 1 to 4 on the panel of the model shown in Fig. 1. The loading configurations 1 and 2 mainly involve bending of the horizontal elements, while shear type deformations correspond to configurations 3 and 4. The selected loading configuration is transmitted by the user to a computer by pushing one of the four buttons available at the base of the physical model.

Selected loading configurations and applied loads are acquired with a NI CompactDAQ 9172 system by National Instrument ©, interfaced with LabVIEW 8.5.1 (National Instruments ©) and transmitted to a computer.

Using the pre-programmed solutions obtained from linear elastic theory for inextensible rods (see Appendix A), the computer calculates in real time the current deformed shape of the frame, receiving as inputs the loading configur-ation and the applied load.

The theoretical deformed shape is displayed immediately on a monitor (together with the measured load), which can be compared to that of the physical model (see the sequence of photos in Figure 3). Moreover, in order to provide a reference during the comparison, the undeformed configuration of the frame remains visible on the monitor and is also drawn in the background of the model. It may also be interesting to note that the calculation of the frame is so fast that it changes following the oscillations of the load induced by hand loading. This produces an interesting effect, in which the user feels the effect of ‘interacting directly with the theoretical result’.

3. FEEDBACK

The presented model has been used as a teaching tool during ordinary classes of Structural Mechanics and Statics at the University of Trento for undergraduate students of Civil and Mechanical Engineering. As reported in the assessment questionnaires, the students there was felt a simplification of the learning process. As they were playing with the demonstration tool, mechanical concepts (as bending moment, strain energy, unbraced frame) taught during classes became clearer. Improvement of their mechanical intuition was assessed during the final exam of the course.

The model has also been used during public demonstrations (among others, at the so-called ‘researchers' night’ and at university orientation courses organized by the ‘Scuola Normale Superiore’ of Pisa). These experiences have

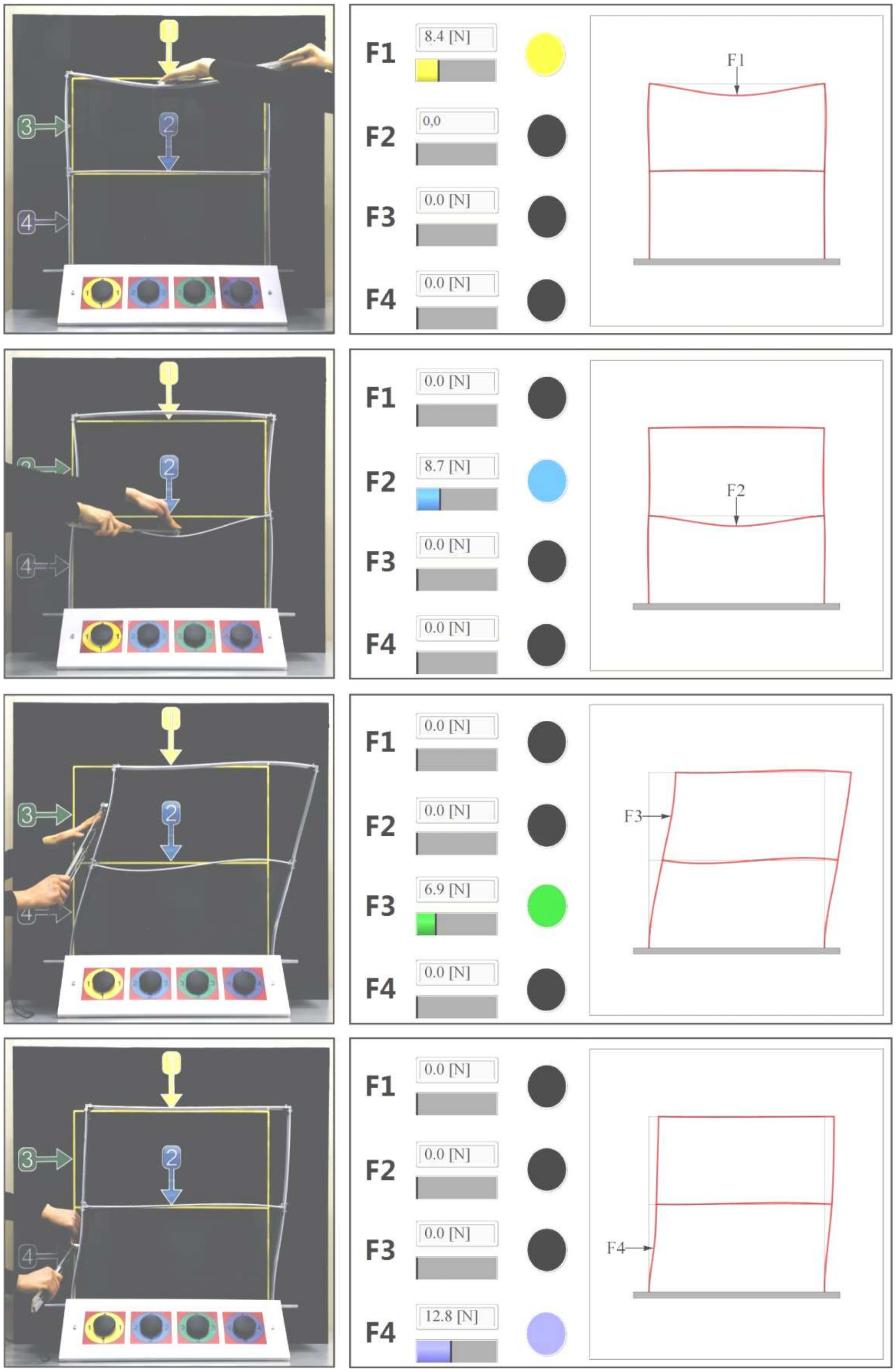

Figure 3. Example of use of the teaching model. The four configurations loaded by the user, from the upper to the lower part: Configuration 1 with F1=8.4N, Configuration 2 with F2=8.7N, Configuration 3 with F3=6.9N, and Configuration 4 with F4=12.8N. During use, the deformed shapes of the frame (left) can immediately be compared with the theoretical results predicted from a linear elastic calculation and displayed on the monitor (right).

shown a notable enthusiasm of young users and of untrained people for the demonstration tool. This is fundamental to create positive public opinion for Mechanical Engineering and to induce school children to think about mechanical concepts with the hope that they will be drawn to engineering at university level.

4. CONCLUSIONS

A teaching model was designed, constructed and tested to facilitate understanding of the deformation of elastic structures with elements subject to bending. The model allows the direct comparison between results of a linear elastic calculation and the real deformation of a structure. This model has proven to be effective not only for teaching purposes, but also to demonstrate to a broad and untrained audience how engineering calculations can be predictive of the observed behaviour.

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APPENDIX A

THE PRE-PROGRAMMED SOLUTIONS OBTAINED FROM LINEAR ELASTICITY

With reference to the Euler-Bernoulli inextensible beam model, the internal bending distribution $M_i(s_i)$, along the longitudinal coordinate s_i within the *i*-th span, is related to the curvature $\chi_i(s_i)$ through the linear elastic constitutive equation

$$M_i(s_i) = EJ_i\, \chi_i(s_i)$$

where EJ_i is the bending stiffness, given as the product of the Young's modulus E (=2350 MPa for solid polycarbonate) and the Inertia Moment $J = bh^3/12$ (=213 mm^{-4} for the considered cross section), where b (=40 mm) is the width and h (=4 mm) is the thickness. Under the assumption of small displacement, the curvature $\chi_i(s_i)$ can be approximated by the second derivative of transverse displacement, namely

$$\chi_i(s_i) = y_i''(s_i)$$

so that, double integration of the bending moment $M_i(s_i)$ and the imposition of displacement and rotation conditions at the joints and constraints, lead to the displacement field $y_i(s_i)$ describing the deformed shape. Due to the presence of concentrated loads, the bending distributions are linear and therefore the deformed shapes are defined as cubic functions.

The bending moment distribution within the six spans of the statically indeterminate structure reported in Fig 1 can be obtained using one of the several methods available in linear elasticity (among others, Energy method and Matrix force method). Since linear elasticity implies superposition principle, the solution can be obtained from the four fundamental solutions for the unit force applied to each of the four application load points.

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