Homogenization-based Constitutive Model for Periodically Fiber-Reinforced Elastomers

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When compared to more conventional structural materials fiber-reinforced polymeric materials are characterized by superior stiffness and strength-to-weight ratio, as well as excellent fatigue and creep resistance over a broad range of temperatures. In addition, fiber-reinforced-type morphologies appear naturally in a number of soft matter systems of increasing interest such as thermoplastic elastomers, or soft biological tissues like arterial walls, ligaments, and the annulus fibrosus of the human intervertebral disc.

A homogenization-based constitutive model [1] is presented for the mechanical behavior of hyperlastic elastomers reinforced with aligned cylindrical fibers subjected to finite deformations. The model incorporates full dependence on the constitutive behavior of the constituents (i.e., the matrix phase and the fibers). Furthermore, the model accounts for statistical information about the initial microstructure beyond the volume fraction, as well as for its evolution, which results from the finite changes in the geometry that are induced by the applied finite deformations, and can have a softening or hardening effect on the overall response, leading to the possible development of macroscopic instabilities.

The proposed model is derived by making use of the second-order homogenization method [2], which is based on suitably designed variational principles utilizing the idea of a *linear comparison composite*. Specific results are generated for the case when the matrix and fiber materials are characterized by Gent solids, with stored-energy function

$$W = -\frac{\mu J_m}{2}\ln\left(1-\frac{I-3}{J_m}\right) - \mu \ln J + \left(\frac{\kappa}{2} - \frac{3+J_m}{3J_m}\mu\right)(J-1)^2, \tag{1}$$

and the distribution of fibers is periodic, with square and hexagonal arrangement (see Fig. 1).

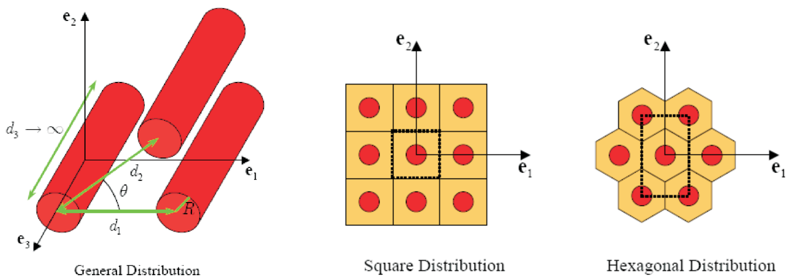

Figure 1: Schematic representation of the microstructures in the undeformed configuration.

The elastomeric composite exhibits orthotropic material symmetry, and hence, distinguishes between fiber- and matrix- dominated modes of deformation. Loading conditions that involve deformation in the fiber direction lead to much stiffer model responses than those that do not induce fiber deformation. For fiber-dominated modes (Fig. 2), the behavior of the fiber-reinforced elastomers is controlled primarily by the fiber volume fraction and the contrast between the matrix and fiber phases. For matrix- dominated modes of deformation (Fig. 3), the overall material behavior is controlled by the in-plane distribution and volume fraction of the fibers, provided that the fibers are sufficiently stiffer than the elastomeric matrix phase.

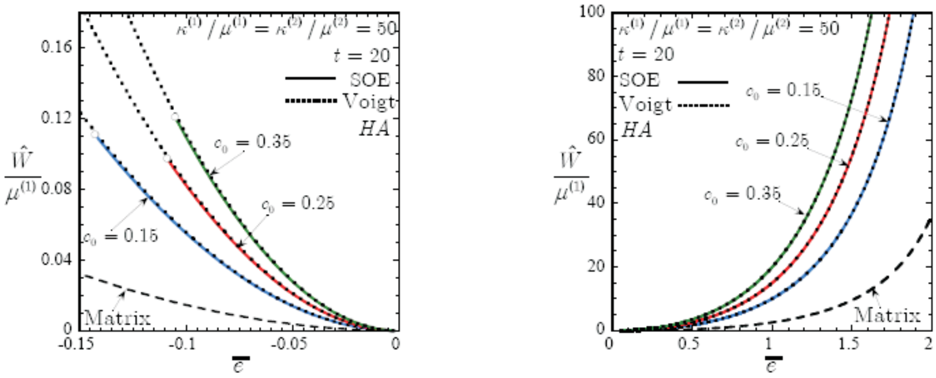

Figure 2: Effective response of a fiber-reinforced elastomer subjected to axisymmetric shear $\overline{\mathbf{F}} = \overline{\lambda}^{-1/2}(\mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2) + \overline{\lambda}\mathbf{e}_3 \otimes \mathbf{e}_3$. The normalized effective stored-energy function $\widehat{W}/\mu^{(1)}$ is given as a function of the logarithmic strain $\overline{e} = \ln \overline{\lambda}$ for various initial volume fraction c_0.

For compressive loadings in the fiber direction, the derived constitutive model may lose strong ellipticity, indicating the possible development of macroscopic instabilities that may lead to kink band formation. The onset of shear band-type instabilities is also detected for in-plane modes of deformation in a way that is consistent with the breaking of the symmetries of the underlying, highly ordered, periodic microstructure.

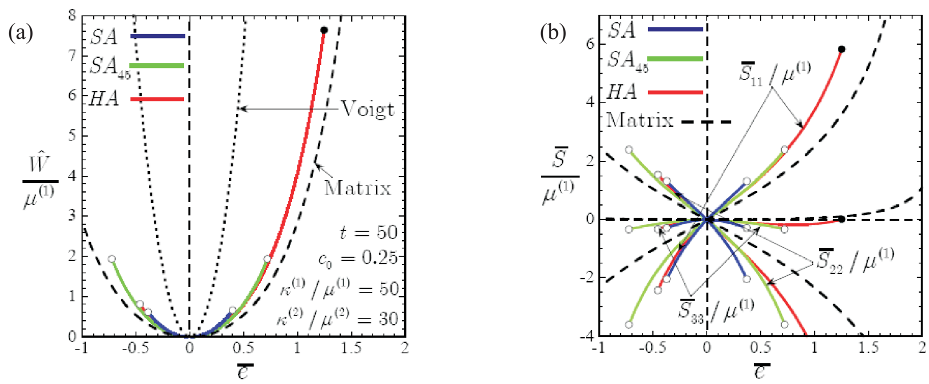

Figure 3: Effective response of a fiber-reinforced elastomer subjected to in-plane pure shear $\overline{\mathbf{F}} = \overline{\lambda}\mathbf{e}_1 \otimes \mathbf{e}_1 + \overline{\lambda}^{-1}\mathbf{e}_2 \otimes \mathbf{e}_2 + \mathbf{e}_3 \otimes \mathbf{e}_3$ for periodic square (SA) and hexagonal (HA) arrangement of fibers. The normalized effective stored-energy function $\widehat{W}/\mu^{(1)}$ (a) and the normalized stress components $\overline{S}_{11}/\mu^{(1)}$, $\overline{S}_{22}/\mu^{(1)}$ and $\overline{S}_{33}/\mu^{(1)}$ (b) are given as a function of the logarithmic strain $\overline{e} = \ln \overline{\lambda}$.

References

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[2] *O. Lopez-Pamies and P. Ponte-Castañeda*. On the overall behavior, microstructure evolution, and macroscopic stability in reinforced rubbers at large deformations: I-Theory. *J. Mech. Phys. Solids*, Vol. 54, pp. 807-830, 2006.