FIBER-REINFORCED ELASTOMERIC MATERIALS AT FINITE DEFORMATIONS: EFFECTIVE BEHAVIOR AND MACROSOPIC STABILITY

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1 INTRODUCTION

Elastomeric composites are widely used in industry (tires, shoes, cable coatings, balloons, shock absorbers, etc.), often in situations involving large deformations. Two sources of nonlinearities make the constitutive modeling quite difficult: first, the single phases present strong material nonlinearity; second, the finite changes in the geometry induce an evolution of the microstructure in terms of volume fraction, shape, position and orientation of the fillers, which affects the overall behavior.

In the present work, estimates are presented for the overall behavior of two-phases, fiber-reinforced, rubber composites subjected to large deformations by means of the recently developed "second-order" homogenization method (SOM) of Ponte Castañeda (2002 a,b), extended to hyperelastic composites by Lopez-Pamies and Ponte Castañeda (2004).

The central idea of these powerful technique is the use of the fictitious *linear comparison composite*, which is chosen through suitably designed variational principles and, as demonstrated by Ponte Castañeda (2002 a,b), is able to carry information not only about the average deformation fields in the phases but also about the local field fluctuations.

We underline the fact that the method is quasi analytical and it simply reduces to a solution of a nonlinear system of algebraic equations, which make it suitable for a future implementation in standard finite element codes in order to perform full structural analysis.

2 EXAMPLES

Applications are considered for periodic hexagonally packed and square microstructure subjected to generalized plane strain macroscopic deformations where the plane of deformation is orthogonal to the fiber direction $\mathbf{x}_3$.

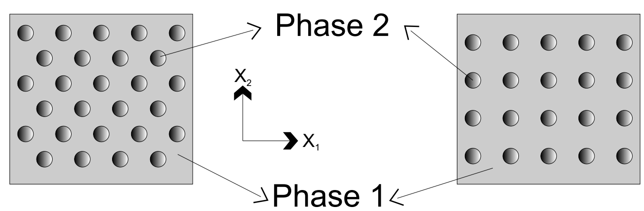


Figure 1: Hexagonally packed and Square array microstructure

2.1 Axisymmetric loadings: the Voigt bound

Fig. (2) shows the effective stored-energy function for incompressible phases subjected to axisymmetric macroscopic deformation: $\bar{e}_3 = e$, $\bar{e}_1 = \bar{e}_2 = -e/2$, where $e = \ln(\lambda)$ is the logarithmic strain. In particular, each phase is modeled as a compressible Neo-Hookean material with stored-energy function:

$$W(\mathbf{F}) = \frac{\mu}{2}(\mathbf{F}\cdot\mathbf{F} - 3) - \mu \ln J + \left(\frac{\kappa}{2} - \frac{\mu'}{3}\right)(J-1)^2,$$

with μ and κ denoting the shear and the bulk moduli and $J = \det \mathbf{F}$.

It is clearly seen from fig. (2) the stiffening effect produced by the addition of the stiffer fibers into the elastomeric matrix.

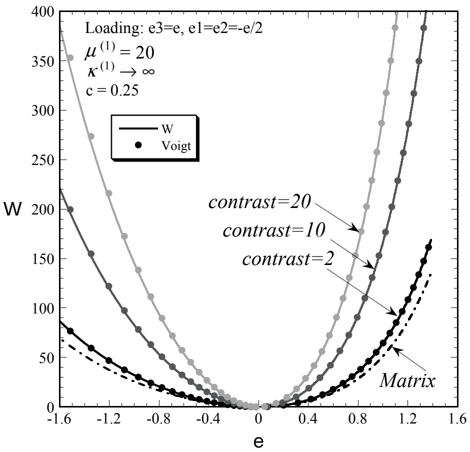
Under axisymmetric loading conditions, the exact deformation field in the case of incompressible phases is in fact uniform, and consequently the macroscopic behavior is given by the Voigt bound, which is in agreement with the second-order estimates.

2.2 Pre-compressed simple shear

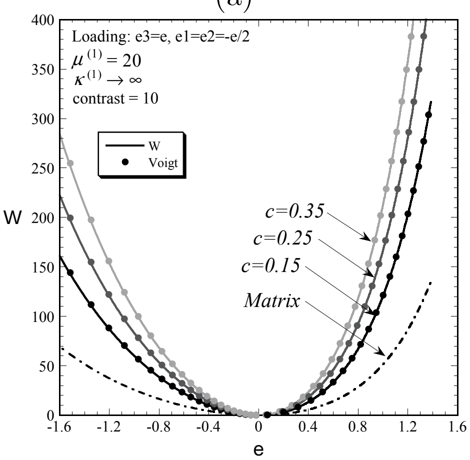
In fig. (3) the shear component S_{12} of the first Piola-Kirchhoff stress is shown as a function of the amount of shear deformation γ; a pre-compression in direction x_2 is also applied so that the composite is subjected to the overall deformation gradient

$$\overline{\mathbf{F}} = \mathbf{I} - 1/2\,\mathbf{e}_2 \otimes \mathbf{e}_2 + \gamma\,\mathbf{e}_1 \otimes \mathbf{e}_2.$$

The second-order estimates for the macroscopic stress show an oscillation which has been recently observed by Lahellec et al. (2004) and which is related to the evolution of the microstructure from a configuration in which the fibers are aligned and one in which they are arranged crosswise and viceversa. The macroscopic behavior depends strongly on the microstructure evolution.



(a)



(b)

Figure 2: Second-order estimates for the effective behavior of hexagonally packed reinforced elastomers subjected to the macroscopic loading $\bar{e}_3 = e$, $\bar{e}_1 = \bar{e}_2 = -e/2$. The results are given as a function of the logarithmic strain e for Neo-Hookean phases with bulk modulus $\kappa^{(1)} = \kappa^{(2)} \to \infty$. In fig. (2a) the results correspond to contrasts $\mu^{(2)}/\mu^{(1)} = 2, 10, 20$ for fixed initial volume fraction $c_0 = 0.25$. In fig. (2b) the results are given for initial volume fractions $c_0 = 0.15, 0.25, 0.35$ at fixed contrast $\mu^{(2)}/\mu^{(1)} = 10$. The continuous lines correspond to the second-order estimates, the dashed lines to the Voigt bounds and the dashed-dotted line to the matrix behavior.

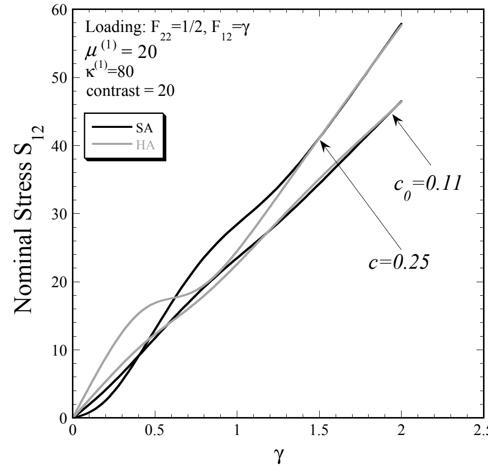


Figure 3: Second-order estimates for the effective behavior of elastomers reinforced with long fibers subjected to pre-compressed simple shear in the plane orthogonal to the fiber direction. The results are given as a function of the shear strain γ for compressible Neo-Hookean phases with $\mu^{(1)} = 20$, bulk modulus $\kappa = 4\mu$, contrast $\mu^{(2)}/\mu^{(1)} = 20$ and initial fiber concentrations $c_0 = 0.11, 0.25$. The hexagonally packed and the square periodic distributions are reported in gray and black, respectively.

2.3 Macroscopic stability

As indicated by Geymonat et al. (1993) the onset of macroscopic instabilities can be identified with the loss of strong ellipticity of the homogenized constitutive behavior which has been estimated with the second-order method.

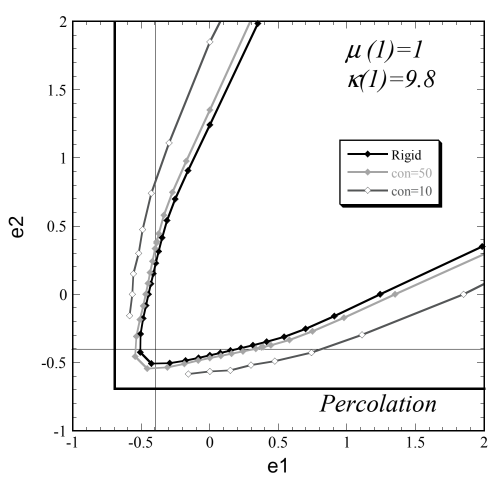


Figure 4: Loss of ellipticity of the nonlinear composite in the deformation space $(\mathbf{e_1}, \mathbf{e_2})$. The results are given for compressible Neo-Hookean phases with $\mu^{(1)} = 1$, $\kappa^{(1)} = 9.8$, and contrasts $\mu^{(2)}/\mu^{(1)} = \kappa^{(2)}/\kappa^{(1)} = 10, 50, \infty$. The fiber are distributed with square periodicity and initial concentration $c_0 = \pi/16$.

Fig. (4) displays the strongly elliptic domain for elastomers with square distributions of fibers. The results are shown in the logarithmic strain plane $(\mathbf{e_1}, \mathbf{e_2})$ for the three different contrasts $\mu^{(2)}/\mu^{(1)} = \kappa^{(2)}/\kappa^{(1)} = 10, 50, \infty$.

REFERENCES

G. Geymonat, S. Muller, S. Triantafyllidis, Homogeneization of nonlinearly elastic materials, macroscopic bifurcation and macroscopic loss of rank-one convexity, *Arch. Rat. Mech. Anal.*, **122**, (1993), pp. 231–290.

N. Lahellec, F. Mazerolle, J.C. Michel, Second-order estimates of the macroscopic behavior of periodic hyperealstic composites: theory and experimental validation, *J. Mech. Phys. Solids*, **52**, (2004), pp. 27–49.

O. Lopez-Pamies, P. Ponte Castañeda, Second-order estimates for the macroscopic response and loss of ellipticity in poroous rubbers at large deformations , *J. Elasticity*, **76**, (2004), pp. 247–287.

P. Ponte Castañeda, Second-order homogenization estimates for nonlinear composites incorporating field fluctuations. I. Theory, *J. Mech. Phys. Solids*, **50**, (2002), pp. 737–757.

P. Ponte Castañeda, Second-order homogenization estimates for nonlinear composites incorporating field fluctuations. II. Applications, *J. Mech. Phys. Solids* **50**, (2002), pp. 759–782.