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Damage Localization in Reinforced Concrete Structures by using Damping Measurements

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Abstract. The vibration measurement approach to damage localization generally consists in correlating dynamic quantities, which detect the presence of damage globally, with damage indicators, which model the damage locally. Even if no restriction is imposed theoretically on the choice of such quantities and indicators, it is a fact that the methods proposed in the literature are almost always based on the measurement of natural frequencies or modal shapes and modeling of damage as local reduction in stiffness.

A theoretical-experimental research project has been in progress for many years at the University of Padua. It is aimed at evaluating the possibility of using dynamic testing to identify manufacturing defects or structural damage in precast reinforced concrete elements. Experimentally, the research involves the dynamic characterization of a set of hollow panels, 1.20m x 5.80m wide, both undamaged and cracked. Characterization techniques are based on stepped sine tests and shock tests.

Early results of experiments show how the presence of a small, visually undetectable crack causes negligible variations in natural frequencies, but a considerable increase in damping. In some cases variations in damping rate to the order of 50% were measured.

This paper describes how damping variation can be related to the formation of local hysteretic mechanisms, and refers to Bachmann's studies as a starting point for damage modeling. Two new localization techniques, based on damping measurements, are consequently proposed. The first is simply based on the variation of an energetically equivalent modal damping ratio. A second approach consists in distinguishing the contribution of a viscous and friction component in damping, and identifying the damage through friction detection and characterization. Joint time frequency analysis is used to extract the parameters of interest.

These techniques are applied to damage localization in a cracked hollow panel, and the results are compared with the outcomes of frequency and curvature based approaches.

Introduction

Since 1979, when Cawley and Adams [1] first proposed a technique for damage localisation based on vibration measurements, many new theoretical methods and few experimental results have been presented, but no actual applications with truly satisfactory outcomes. This is particularly true in the field of civil engineering, thus the methodological approach to damage localisation deserves some consideration.

Damage localisation methodology. It is possible to summarise the damage localisation procedure in the following logical sequence, which is always implicit in the methods proposed in the literature:

1. individuation of dynamic measurements that identify damage at a global level (e.g. variation of modal parameters, non linearity indices...);
2. modelling of damage at a local level and definition of the local indicators δ (e.g. as local variations in stiffness, structural discontinuity, local variations in viscosity);
3. choice of a relation that connects global measures z to local damage parameters δ, $z = F(\delta)$;
4. calculation of local damage parameters by resolution of the inverse problem.

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Of course no restriction on the choice of such measures, which detect damage at a global level, is theoretically imposed. Nor is it imposed on the choice of local damage indicators. Nevertheless, methods proposed in literature are almost always based on the measurement of natural frequencies or modal shapes (in terms of displacement or more recently in terms of curvature [2]) and the modeling of damage as a local reduction in stiffness.

It is also important to distinguish between techniques based on identification of variations in response, generally expressed in the classical modal parameters (frequency, shape, damping). These regard the configuration of a known, and conventionally intact model, and techniques based on the identification of anomalies, usually non-linearity. In the first case damage is revealed by a difference in behaviour of two structures which, on the basis of response only, cannot be declared undamaged or damaged. In the second case it is the presence of the anomaly itself that identifies damage. It is worth noting that the second approach is much less frequent in proposals for damage detection, and it would appear that it has never been used in localisation problems.

Damage detection in RC and PRC structures. The dynamic behaviour of cracked and uncracked RC has been investigated in the past in many publications [3,4,5,6,7,8]. Authors, starting with Bachmann, have given their attention to damping variation and the formation of non-linear dissipation mechanisms together with cracking. The peculiarity of dynamic responses of RC structures should justify a major interest in damping and non-linearity identification.
The aim of this paper is to show how an approach to damage identification is possible, as an alternative to the usual modelling of damage as a reduction in stiffness. Hence, outcomes of an experimental research program underway at the University of Padova, on components of precast reinforced concrete are used. This research is in response to the need to verify and assess the integrity of industrial production as well as existing structures and is aimed at evaluating the possibility of using dynamic tests to identify construction defects or damage in RC and PRC structures. The experiment conducted on a precast hollow floor panel is briefly reported here. Further details can also be found in [9,10].

Experimental research

The geometry of the specimen, 5.8 m long and 1.2 m wide, and measurement positions are shown in Fig. 1. The structure was characterised dynamically before and after introduction of localised damage. On the basis of a critical analysis of data supplied by a previous experiment [9] all tests were conducted using an isostatic three point bearing system. Damage was introduced to the structure by applying a concentrated load in correspondence to one third of the span with monotonic increasing intensity until the opening of a crack, which closed again after unloading. The damage introduced can be detected only by instrumental measurement, and its presence indicates that a limit state is reached (overloading or reduced prestressing).

Dynamic characterisation. The dynamic characterisation of the panel before and after cracking was carried out using both stepped-sine tests and shock tests. Harmonic excitation was applied to the centre, using a harmonic exciter with a single rotating mass, in a frequency range 1 to 40 Hz. Impulsive excitation was applied to the structure by using a pulse hammer at the most important position in order to excite the main mode shapes. Figs. 2 and 3 show some examples of the FRF achieved, before and after damage.

Extraction of modal parameters. The modal parameters of the structure before and after the introduction of damage were extracted from acquired signals by using different techniques.

Natural frequencies and modal components were found by MDOF fitting of the experimental FRFs. In Table 1, frequencies of corresponding identified modes of both the undamaged and cracked panels are compared; differences are very small.

Table 1: identified frequencies and modal recognition

frequency [Hz] undamaged	frequency [Hz] cracked	mode
19.55	19.27	I X-bending mode
20.84	20.52	I torsional mode
42.98	41.93	I Y-bending mode
59.84	59.67	II X-bending mode
68.40	67.02	II torsional mode

The extraction of damping rate was carried out using different techniques: MDOF fitting of the FRFs; evaluation of the decay in oscillation amplitude by using modal filtering and Hilbert Transform (HT), after Feldman [11]; evaluation of the decay in oscillation amplitude by using Short Time Fourier Transform, after [12]. In each case, the hypothesis of linear viscous damping was assumed - i.e. an exponential decay of the amplitude time history was considered - both before and after the appearance of cracking.

As shown in Table 2, modal damping rates vary greatly following cracking, particularly in correspondence with the first X-bending mode.

Table 2. Damping ratios before and after damage obtained by various extraction techniques.

identified mode	damping rate ξ [%] MDOF curve fitting		through HT		through STFT	
	undamaged	cracked	undamaged	cracked	undamaged	cracked
I X-bending mode	0.58	1.12	0.42	1.17	0.45	1.07
I torsional mode	0.62	0.56	0.56	0.59	0.58	0.58
I Y-bending mode	1.00	1.14	0.79	0.96	0.85	0.96
II X-bending mode	0.86	0.88	0.84	0.95	0.86	0.86
II torsional mode	1.10	1.27				

General approach to damage localization in the case study. The general idea is to divide the panel into a certain number of elements (5 or 9), which are homogeneous in the damage parameter (stiffness, specific damping...). A linear function (*sensitivity matrix*), is assumed as the relation that links measures to parameters. The different damage models assumed, the consequential localisation techniques applied and outcomes, are described in detail in the following sections.

Damage as a local variation in stiffness

Formulation. This is the common approach to damage modelling: a system with frequencies and mode shapes is given:

$$\mathrm{M}\ddot{x} + \mathrm{K}x = 0 \quad \Rightarrow \quad \phi_1 \ldots \phi_n \quad \omega_1 \ldots \omega_m . \tag{1}$$

Damage is modelled as a variation in stiffness $\Delta\mathrm{K}$, which is a linear function of a vector of parameters δ_i that quantify damage at a local level (damage indicators):

$$\Delta K = \delta_1 K_1 + \delta_2 K_2 + \ldots + \delta_n K_n . \tag{2}$$

Variations of sth frequency and mode shape are related to damage in the following expression:

$$(K + \Delta K)(\phi_s + \Delta\phi_s) = (\omega_s^2 + \Delta\omega_s^2)M(\phi_s + \Delta\phi_s), \tag{3}$$

which is typically non linear.

Frequency variation method. Eq. 3 could be linearised in:

$$\Delta\omega_s = \sum_i s_{si}\delta_i , \tag{4}$$

where s_{si} is the sensitivity of the sth frequency to damage in i. However the sensitivity matrix is calculated, the frequencies were found not to be very sensitive parameters to damage, and localisation did not give good results (Fig. 4).

Modal curvature method. The technique, initially proposed empirically in [2], could also be derived from Eq. 5 [10]; the damage indices in the case study became:

$$\delta_i = \frac{\Delta\chi_{is}}{\chi_{is}}, \tag{5}$$

where χ_{is} represents the bending curvature of the sth mode, in the ith portion of the panel. The method, applied to the differences between curvatures, between the theoretical trend of curvature before cracking and the experimental trend after cracking of the first mode shape, has given a precise localisation of damage (Fig. 5). This is detected around the actual position of cracking with an indeterminacy (function of the number of measurement points) of 72 cm.

Damage as a local variation in specific damping

The high sensitivity to damage shown by modal damping rates in experimental testing, suggests utilising this measure in damage localisation.

Formulation of modal damping variation method. The damping ratio can be expressed in the case of a classically damped system as

$$\xi_s = \frac{\omega_s}{2}\frac{\phi_s^+ C\phi_s}{\phi_s^+ K\phi_s}, \tag{6}$$

but more generally it can be interpreted as a ratio of dissipated energy in a cycle and the maximum potential energy of the system [4]:

$$\xi_s = \frac{\Delta E_D}{4\pi E_P} = \frac{\Delta E_D}{2\pi\phi_s^+ K\phi_s} . \tag{7}$$

An analogous expression can be written, with reference to the vibration mode, for an ith portion of the vibrating system:

$$\xi^{(i)} = \frac{\Delta E_{D,i}}{2\pi\phi_s^+ K_i\phi_s} \tag{8}$$

and in this case $\xi^{(i)}$ assumes the meaning of *specific damping ratio*.

Given a complete group of subsystems, for each vibration mode it is necessary that:

$$\Delta E_D = \sum_i \Delta E_{D,i} \tag{10}$$

for which:

$$\xi_s = \sum_i \frac{\phi_s^+ K_i \phi_s}{\phi_s^+ K \phi_s} \xi^{(i)} \tag{11}$$

The presence of a crack in the *i*th element of the system generally leads to the formation of a local dissipative mechanism which induces an increase in specific damping, and therefore the local damping ratio. In the simplified hypothesis where damage does not degrade the stiffness of the system, one can write:

$$\Delta \xi_s = \sum_i \frac{\phi_s^+ K_i \phi_s}{\phi_s^+ K \phi_s} \delta_i \tag{12}$$

where $\delta_i = \Delta \xi^{(i)}$ represents the damage index.

Application to the case study. The total damping variation for each bending mode can be expressed as:

$$\Delta \xi_s = \frac{\sum_i \delta_i \chi_{is}^2}{\sum_i \chi_{is}^2}, \tag{13}$$

and is therefore linked to the square of the curvature of the mode shape. For the torsional modes Eq. 13 becomes:

$$\Delta \xi_s = \frac{\sum_i \bar{\delta}_i \vartheta_{is}^2}{\sum_i \vartheta_{is}^2}, \tag{14}$$

and therefore a link with the square of the rotation (i. e. of the derivative) of the mode shape is hypothesized. Given the anisotropy of the damage, it is reasonable to believe that the variation in specific damping appears to be different in the case of bending, compared with torsional modes. It can be hypothesised, therefore, that dissipation due to torsional vibration is proportional to that for bending vibration by means of the index k, also to be identified:

$$\bar{\delta}_i = k \delta_i \, . \tag{15}$$

Eqs. 13 and 14, combined with Eq. 15, represent a non linear algebraic system, in k and δ_i, that can be resolved iteratively. The modal parameter is particularly sensitive to damage and despite the indeterminacy of results due to symmetry, damage is detected in two bands of 145 cm width around the actual cracking (Fig. 6).

Damage in the presence of a friction mechanism.

SDOF oscillator with Coulomb friction. The presence of dry friction in a SDOF system can be represented by a hysteretic dashpot. This reacts with a restoring force which is constant in amplitude and depends only on the direction of motion:

$$F = -F_C \frac{\dot{x}}{|\dot{x}|}. \tag{16}$$

Therefore, the mass normalised equation of motion of the system is:

$$\ddot{x} + 2\xi\omega\dot{x} + \omega^2 x_{lim}\frac{\dot{x}}{|\dot{x}|} + \omega^2 x = 0 \tag{17}$$

where the *limit displacement* is defined:

$$x_{lim} = \frac{F_C}{k} \tag{18}$$

as x_{lim} represents the limit displacement in static equilibrium. Eq. 17 is typically non linear. An exact solution could be obtained through numerical integration [13,14]. For our aims, simple energetic considerations are enough. Fig. 7 shows the typical restoring-force versus displacement diagram for a SDOF oscillator with viscous damping, while Fig. 8 shows a typical hysteretic cycle for a friction damped oscillator. in each case a represents the oscillation amplitude. Referring to the viscous dashpot, the dissipated energy for a cycle is, in the case of sinusoidal oscillation $x = a\,sin\omega t$:

$$\Delta E_v = \int_{\Omega} -F(\dot{x})dx = \int_0^T c\dot{x}\cdot\frac{dx}{dt}\cdot dt = \pi c\omega a^2\ ; \tag{19}$$

in the case of the friction device the dissipated energy gives

$$\Delta E_f = \int -F(\dot{x})dx = 4F_C a\,, \tag{20}$$

and this expression is true whatever the time history of the system, as long as a cycle is characterised by no more than two velocity zero-crossing. In the case of free vibration, i. e. in the absence of any energetic input to the system, the loss of mechanical energy in a cycle must be equal to the dissipated energy:

$$\Delta E_m = \Delta E_v + \Delta E_f \tag{21}$$

Note that in the presence of a Coulomb friction mechanism, free motion is generally non sinusoidal. Anyway, this approximation is widely acceptable when elastic forces are greater than friction forces. In this case, the normalised expression becomes:

$$\Delta\left(\frac{1}{2}\omega^2 a^2\right) = \pi 2\xi\omega^2 a^2 + 4\omega^2 x_{lim} a \tag{22}$$

where a now stands for an intermediate value of the amplitude of oscillation, inside the cycle.

The same balance can be expressed in terms of power, dividing each member by $\Delta t = T = \frac{2\pi}{\omega}$, and obtaining:

$$\frac{da}{dt} = \xi\omega a + \frac{2}{\pi}\omega x_{lim} \tag{23}$$

Let us define the *friction amplitude*:

$$a_f = \frac{2}{\pi \cdot \xi} \cdot x_{lim} \tag{24}$$

so that the previous expression simplifies to:

$$\frac{da}{dt} = \xi\omega(a + a_f) \tag{25}$$

which integral is:

$$a(t) = (a_0 + a_f) \cdot e^{-\xi \cdot \omega \cdot t} - a_f \tag{26}$$

Formulation of the modal friction damping method. Since Eq. 17 is non-linear, modal analysis theory is not strictly applicable in the case of a MDOF system in the presence of Coulomb friction. In any case, it is possible to assume, with negligible error, that mode shapes do not change in free vibration. Therefore, using the same approach as the previous section, it results:

$$(x_{lim})_s = \sum_i \frac{\sqrt{\phi_s^+ K_i \phi_s}}{\phi_s^+ K \phi_s} \delta_i \tag{27}$$

where $(x_{lim})_s$ represents the *s*th *modal limit displacement*, i. e. the limit displacement relative to the free vibration of the system according to the *s*th mode.

Application to the case study. In this case Eq. 27 can be rewritten, for torsional and bending modal shapes, with expressions similar to Eqs. 13 and 14, already extracted in the case of the modal damping variation method.

Modal limit displacement can be extracted from a free monofrequency signal. The procedure is the same as that described in [12], already applied when extracting modal damping in the previous section. It consists in finding the best fitting of the analytic signal amplitude, this time using Eq. 26 instead of a simple logarithmic line, and assuming $(a_f)_s$ and ξ_s as parameters. These immediately go back to $(x_{lim})_s$ keeping in mind Eq. 24. Figure 10 shows the results of fitting applied to the free response of the panel according to the first bending frequency, before and after cracking. It can be noted how the hypothesis for the damaged panel of purely viscous damping (exponential fitting) is rather imprecise. On the other hand, the decay in signal amplitude is very well described in Eq. 26. With the aim of extracting modal limit displacements, it is possible to demonstrate that modal overlapping cannot be considered valid, not even approximately. This is a direct consequence of the non linearity of Eq. 17. In other words, the *s*th modal limit displacement must be extracted from the free response of the panel according to the *s*th vibration mode only. The use of modal filters leads to underestimating the limit displacement relative to higher modes.

In practice, however, obtaining free monofrequency vibration of the panel according to superior modes is technically very complex. Current experimentation is developing in this direction.

Summary

Different approaches to damage detection and damage localisation have been analysed and tested on a PRC structure. Concerning methods based on the interpretation of damage as a local reduction in stiffness, the frequency variation method provides very poor results. On the other hand, the modal curvature method has given satisfactory results, but requires a large number of measurement points and the extraction of at least one mode shape. The proposed method, based on modal damping variations, provides results which are comparable with other techniques. Finally, the use of modal friction damping seems to merit particular attention, even if further experimentation is required in order to extract these parameters in the case of the highest modes. In any case, it highlights the promising possibility of gathering information on the integrity of RC structures by investigating dissipation mechanisms that are formed following damage. It is in this direction that our research is currently developing.

References

[1] P. Cawley and R. D. Adams: *The location of defects in structures from measurements of natural frequencies*, Journal of Strain Analysis Vol. 14(2) (1976), pp. 49-57.

[2] A.K. Pandey, M. Biswas and M. Sammam, *Damage detection from changes in curvature mode shapes*, Journal of Sound & Vibration 145 (2) (1991), pp. 321-332.

[3] A.S.J. Swamidas and Y. Chen: *Monitoring crack growth through change of modal parameters*, Journal of Sound & Vibration 186 (2) (1995), pp. 325-343.

[4] H. Bachmann and R. Dieterle: *Experiments and models for the damping behaviour of vibrating reinforced concrete beams in the uncracked and cracked conditions* (Bericht Nr.119, Institut fur Baustatic und Konstruktion ETH, Zurich, 1981).

[5] G. König and E. Fehling, *Excitation damping of reinforced concrete under cyclic*, XIII IABSE Colloquium, Helsinki (1989), pp. 659-664.

[6] V. Askegaard and H.E. Langsoe: *Correlation between changes in dynamic properties and remaining carrying capacity*, Materials and Structures Vol 19 n.109 (1986), pp.11-20.

[7] O. Mahrenholtz and H. Bachmann: *Comitee Europee du Beton - Bulletin n. 209*, (1991) pp. 169-180.

[8] T. Hop: *The effect of degree of prestressing and age of concrete beams on frequency and damping of their free vibration*, Materials and structures Vol. 24, (1991) pp. 210-220.

[9] G. Beolchini, C. Modena and D. Sonda, *Influenza del danno e di difetti costruttivi sulle caratteristiche dinamiche di pannelli in c.a.p.*, XI congresso CTE, Napoli (1996), pp.65-71.

[10] C. Modena, D. Sonda and D. Zonta: *Sulla localizzazione del danno strutturale in elementi in c.a.p.*, XII congresso CTE, Padova (1998), pp. 293-302.

[11] M. Feldman: *Non linear free vibration identification via the Hilbert Transform*, Journal of Sound and Vibration Vol. 208(3) (1997), pp. 475-489.

[12] C. Valente: *A new procedure for detecting nonlinearity from transient data using the Gabor transform*, Proceeding of 1° Convegno Nazionale di Identificazione Strutturale, Seriate (1995).

[13] G. R. Tomlinson and J. H. Hibbert: *Identification of the dynamic characteristics of a structure with coulomb friction*, Journal of Sound and Vibration Vol. 64(2) (1979), pp. 233-242.

[14] J. A. Inaudi and N. Makris: *Time-domain analysis of linear hysteretic damping*, Earthquake Engineering and Structural Dynamics Vol. 25 (1996), pp. 529-545.

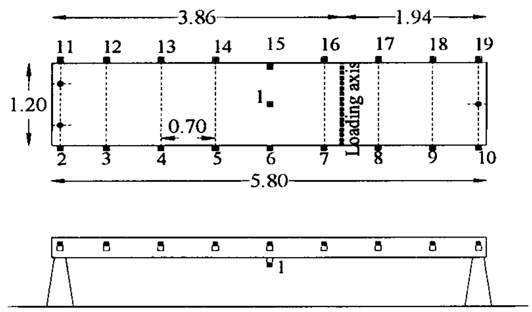

Figure 1. Geometric dimensions of the tested panel and accelerometer position.

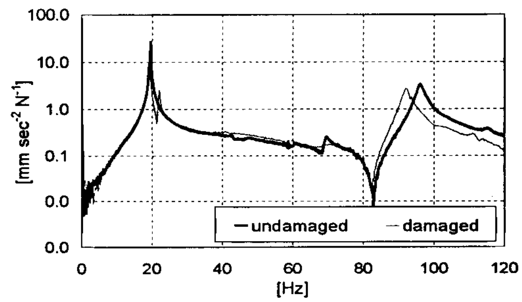

Figure 2. FRF relative to position 1 for an hammer blow in 1.

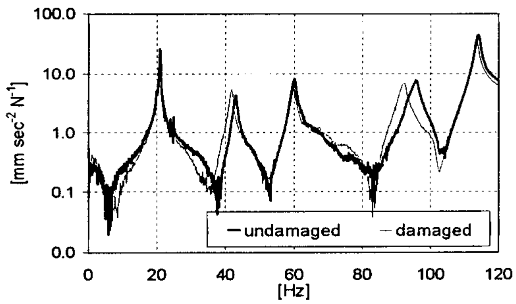

Figure 3. FRF relative to position 8 for a hammer blow in 10.

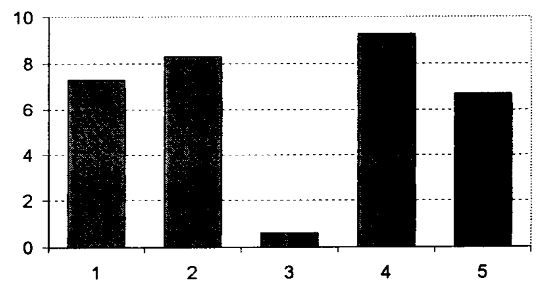

Figure 4. Damage indices calculated by using the frequency variation method. The actual cracking lies in portion 4.

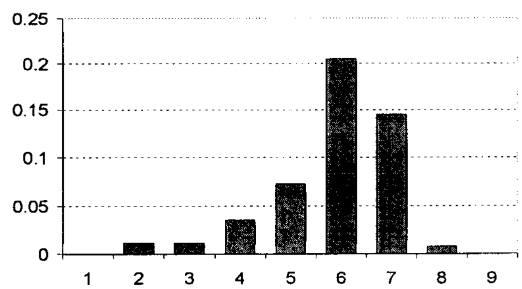

Figure 5: Damage indices calculated by using the modal curvature method. The actual cracking lies between portions 6 and 7.

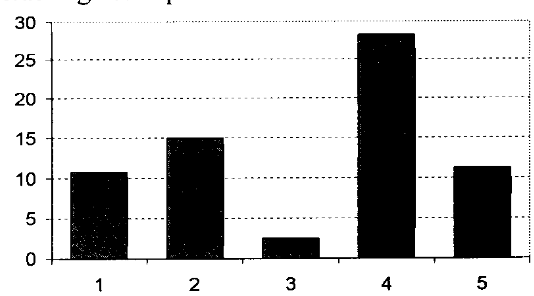

Figure 6: Damage indices calculated by using the modal viscous damping variation method. The actual cracking lies in portion 4.

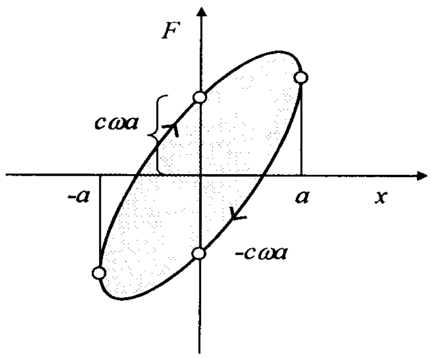

Figure 7. Restoring force relative to an oscillator with viscous damping.

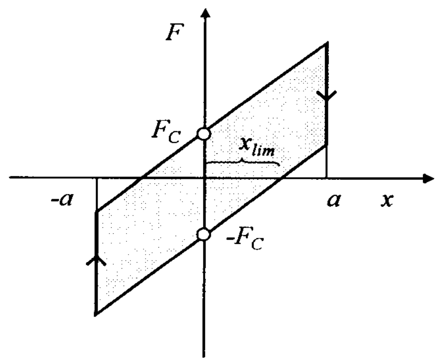

Figure 8. Restoring force relative to an oscillator with Coulomb friction damping.

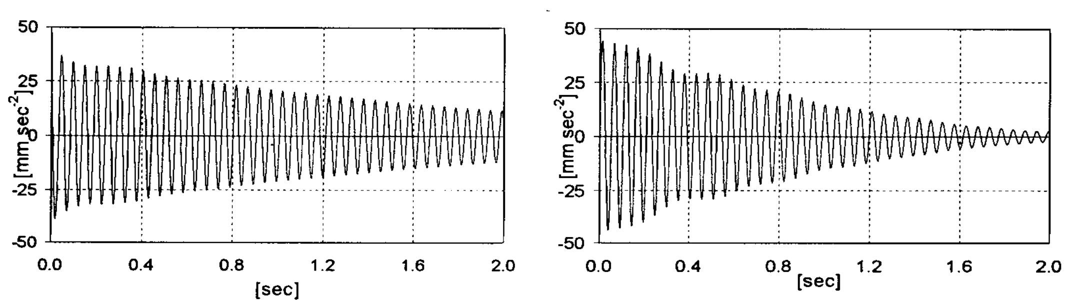

Figure 9. Time history free response, measured in A, of the undamaged (a) and cracked (b) panel: filtered real signal.

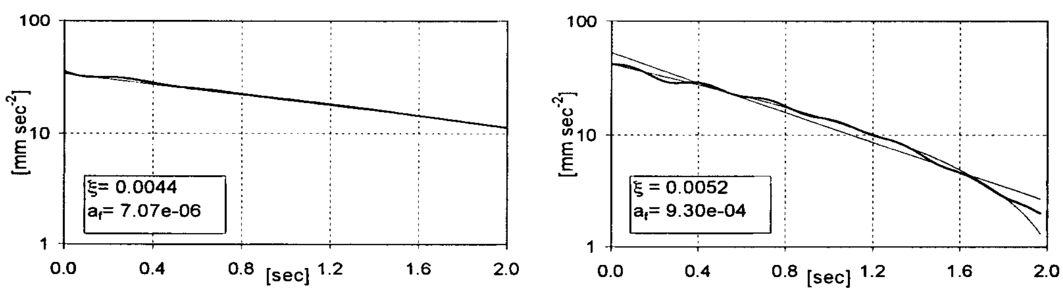

Figure 10. Time history free response, measured in position 1, of the undamaged (a) and cracked (b) panel: amplitude of the analytical signal, exponential fit and non linear fit according to Eq. 26.