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Comparison of two approaches in reliability analysis for the network of Trentino's Bridge Management System

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Abstract. The connectivity reliability of a network states the probability that the traffic can reach the destination from the origin. This paper is dedicated to find a reasonable and fast method to calculate connectivity reliability between any two nodes of a network. Monte Carlo simulation and ORDER algorithm are used and compared in getting the network connectivity reliability. The results show that ORDER algorithm can give more accurate bounds of connectivity reliability for network with high reliable components, while the computation time of Monte Carlo simulation is shorter than ORDER algorithm. Both approaches are applied to the network in the Bridge Management System in the Autonomous Province of Trento.

Introduction

We have already described, in Yue et al. (2010)[1], how the Department of Transportation of Autonomous Province of Trento (APT) is addressing the problem of the seismic vulnerability of its bridge stock. APT manages more than 1000 bridges and approximately 2400 kilometres of roads, through a comprehensive Bridge Management System (BMS). The APT's BMS includes evaluation of seismic vulnerability of each bridge, based on the fragility curve approach, consistent with Hazus guidelines (Federal Emergency Management Agency [FEMA], 2003)[2]. In Yue et al. (2010), we learned that the seismic risk in the APT stock is moderate. However, the system operation at network level is of concern in a post earthquake situation. Approximately 15% of the bridges in the APT stock have a relatively high risk of suffering operational problems. It is therefore necessary to understand the network operation after the earthquake. In this paper, the connectivity between any two given places within the network is calculated. The connectivity reliability of a network states the probability that the traffic can reach the destination from the origin. It is very helpful for bridge managers and government officials in understanding the network status and can assist them to make rapid decisions in near-real time, under post earthquake conditions.

Due to its simplicity and high-speed, Monte-Carlo simulation is often used to calculate the reliability for complex network with a large number of components. However, Monte-Carlo simulation converges slowly when it is used in network with high reliable components. In order to get the accurate bounds of connectivity reliability for network with high reliable components, ORDER algorithm[3] is used to enumerate the most likely states of network. The two approaches are compared in accuracy and computation time. Both approaches are applied to the network in the Bridge Management System in the Autonomous Province of Trento.

The remainder of this paper is as follows. In section 2, we introduce the definition of network reliability using a simple example; in section 3, Monte Carlo simulation and ORDER algorithm are introduced and applied to a simple network; finally, the two methods are applied to the APT network and the results are compared and analysed.

Definition of network connectivity

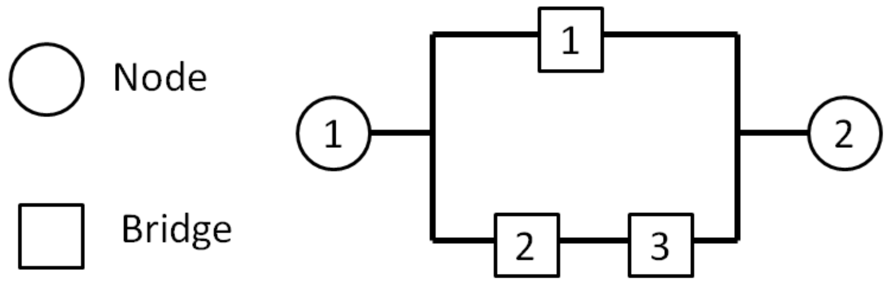

Fig. 1：Simple network 1

Fig. 1 is a simple network with two nodes and three bridges. Bridge i, for i = 1, 2, 3, is in the operational mode with probability p_i, and in the failed mode with probability q_i =1- p_i. The values of p_i (i = 1, 2, 3) are 0.7, 0.8, and 0.9 respectively. In this paper, it is assumed that the bridge elements are the only vulnerable parts of the network, and that the roads between any two bridges will never fail. In this example, we assume that there is no correlation between these bridges; they are all independent of each other. The links between the nodes and the bridges are assumed to be safe. A bridge mode vector V is used to denote the state mode of the bridges: V_i = 1 if bridge i is in failed mode, and 0 if in operational mode.

Table 1. All the network states in Fig.1

Network state	Vector V	Probability of vector V	Connectivity
1	110	0.054	Disconnected
2	101	0.024	Disconnected
3	011	0.014	Connected
4	100	0.216	Connected
5	001	0.056	Connected
6	010	0.126	Connected
7	000	0.504	Connected
8	111	0.006	Disconnected

Given a specific mode vector, if there is at least one path connecting node 1 and node 2, then we say that node 1 and node 2 are connected; otherwise they are disconnected. Table 1 gives all the network states and the corresponding probability for each network state. It shows that there are five network states that are connected. The sum of the probabilities for these five states is 0.916. In this case, we say that the connectivity for the network is 0.916. From this example, we can define the connectivity as the sum of the probabilities of the network states that are connected. In this simple network, there are only 3 bridges and 2 nodes; therefore it is very easy to check the connectivity between two nodes. However, due to the exponential effect, it is difficult to enumerate the state space for a network with more than a few nodes. For a network with 50 bridges, the number of network states is 2^{50} =1.13 $\times 10^{15}$, which is a huge number. In order to solve this problem, two methods are introduced in the following: Monte Carlo simulation and ORDER algorithm.

Monte Carlo simulation and ORDER algorithm

Introduction of Monte Carlo simulation. The Monte Carlo method is also called a statistical simulation method. It is a numerical simulation method using random numbers which are random variables with a uniform distribution in (0, 1). This method was proposed by Metropolis in the Second World War and used in the Manhattan Project. Monte Carlo is the capital city of Monaco, and it is famous for its casino. The basic idea of the Monte Carlo method is to simulate stochastic processes on the computer, and then undertake statistical sampling. Compared with other traditional mathematical methods, it has the advantages of intuitiveness and easy computing.

The Monte Carlo method can be used in many areas. Generally, regarding the characteristics of the stochastic process that it incorporates, the applications of Monte Carlo method can be divided into two types: deterministic problems and random problems. For a deterministic problem, we first built a probability model related to the solution, so that the required solution equals the probability distribution or expectation of the model. We then generate a random variable, and last use the arithmetic mean as the approximation of the solution. Calculating the integral and solving linear equations are associated with this type of problem. For the second type of problem, we normally use a direct simulation method.

Introduction of ORDER algorithm. The Monte Algorithm ORDER was proposed by Li and Silvester (1984), and is used to enumerate the most probable states in a network, *m,* with *n* failure-prone components. Here the failure-prone component means that the probability in operational mode p_i is larger than the probability in fair mode q_i. There are the following assumptions for this algorithm:

1. Component *i* is in the operational mode with probability p_i, and in the failed mode with probability $q_i = 1 - p_i$, all the components are independent.
2. $p_i \geq q_i$.
3. Components are renamed such that $R_1 \geq R_2 \geq \ldots \geq R_n$ where $R_i = q_i / p_i$.

The state of the system is denoted by S_k, $k = 1, 2, \ldots, 2^n$. The probability of S_k is given by

$$P_k = \prod_{i}^{n} p_i (q_i / p_i)^{V_i} \quad (1)$$

where $V_i = 0$ when component *i* is operational, otherwise 1. Obviously, when $V_i = 0$, for $i = 1, 2, \ldots, n$, P_k has the largest value. So the most probable state, S_1, corresponds to no failures, and the next most probable state is the one in which there is only one failed component. This failed component has the largest R_i, i.e., component 1. In this algorithm, the state S_i, $i = 1, 2, \ldots, m$, is identified by the identities of the failed components in S_i ; thus, $S_1 = \varnothing$, $S_2 = \{1\}$, etc.

Let $A = \{S_1, S_2, \ldots, S_m\}$ contains, in decreasing order, the *m* most probable states such that $P(S_1) \geq P(S_2) \geq \ldots \geq P(S_m)$.

Application to a simple network. Fig. 2 is a simple network with 5 nodes and 8 links. The number near each link indicates the probability that the link is in the failed mode, we call it L_i ($i = 1,2,\ldots,8$). Since L_i is very small, this network has high reliable components. Both methods will be applied to this network to calculate the connectivity reliability between node 1 and node 5 in figure 2.

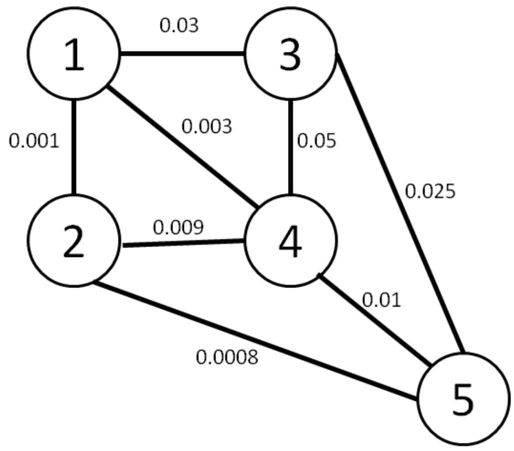

Fig. 2：Simple network 2

The MC method is used as follows:

(1). Generate a random vector X_i ($i = 1,2,\ldots,8$). For each link, if X_i, $< L_i$, then we will delete the link in the network, otherwise keep the link;

(2). Check the connectivity reliability C_i for the remain network, if it is connected, $C_i = 1$, otherwise 0;

(3). Repeat step 1 and 2 for each simulation, If we have m simulations, the final connectivity reliability is $\sum \frac{C_i}{m}$.

The ORDER algorithm is applied as follows:

(1). Generating $A = \{S_1, S_2, ..., S_m\}$: A contains, in decreasing order, the m most probable states such that $P(S_1) \geq P(S_2) \geq \ldots \geq P(S_m)$;

(2). Check the connectivity reliability C_i for each state, if it is connected, C_i =1, otherwise 0;

(3). the final connectivity reliability is $\sum [C_i \times P(S_i)]$.

The results are given in table 2. The calculations are made on a computer with 3.10 GHz CPU. Since there are only 8 links in the network, the total states of the network is $2^8 = 256$, so we only consider the first 100 states with ORDER algorithm. The results show that the connectivity reliability between node 1 and node 5 is 1, we can say that node 1 and node 5 is always connected. Since the network is small, we cannot see the differences between the two methods, next we will apply them to the APT network which is much larger than this one.

Table 2 Connectivity reliability between node 1 and node 5 in figure 2

The value of m		$m = 10^2$	$m = 10^3$	$m = 10^4$	$m = 10^5$	$m = 10^6$
Connectivity	MC	1	1	1	1	1
	ORDER-II	1	-	-	-	-
Computation time(s)	MC	0.012	0.1010	1.019	9.8038	100.704
	ORDER-II	0.0228	-	-	-	-

Application to the APT network

Simulation of APT network. There are 983 bridges in the APT stock, located along SP (province owned) roads and SS (state owned) roads. The whole APT road network, including all bridges and roads, is simulated as a graph. The key phase of network simulation is identifying all the nodes of the graph. The following points are defined as nodes: the intersections or endpoints of SP and SS roads. Each node has 3 variables: ID number, longitude, latitude. Fig. 3 is the simulated graph from Trento to Ala. Trento is the capital city of the APT region, while Ala is an important town in the south of the APT, near the high risk seismic zones in Northern Italy. There are 40 bridges that have different probabilities of being in operational limit state, as represented by the colored dots. SS12 and SP90 are two main roads connecting Trento and Ala. The Adige River and A22 highway are between SS12 and SP90. Only the intersection and the endpoints of SS and SP roads can be identified as nodes. Based on this definition, there are 16 nodes in this graph.

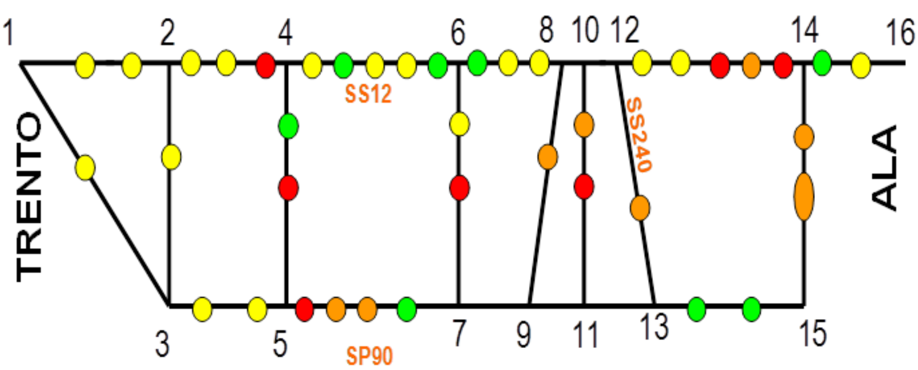

Fig. 3： The graph from Trento to Ala

After identifying all the nodes, the next step is to identify all the links. Not all connections between two nodes can be regarded as links; these must be along the SP or SS roads. There are 22 links in Fig. 3. Every link has 6 variables: ID number, start node ID, end node ID, ID of the road forming the link, the relative position of the start node on the road, and the relative position of the end node on the road. When all the nodes and links are identified, the whole APT network is simulated as a graph in Google Earth as shown in Fig. 4. The small red points represent the nodes, and the red lines represent the links. In total, there are 558 nodes and 740 links in the APT stock. All the bridges are located on the links. Now the algorithms can be performed on the APT network.

Fig. 4： The whole graph

Apply to APT network. In APT network, there are n = 740 links, so the network states is 2^{740}, which is a huge number, it is impossible to enumerates all network states, we can only enumerate the m most probable states, and the approximate total connectivity for this network is:

$$C \approx \sum_{i=0}^{m} C_i \cdot P_i \quad . \tag{2}$$

where P_i is the probability of the i-th network state, and C_i is the connectivity for the i-th network state.

$$C_{2^n} \le C_i \le C_1 \qquad i = 1, 2, \cdots, 2^n \quad . \tag{3}$$

where C_1 is the connectivity when all the links are in operational mode, and C_2^n is the connectivity when all the links are in failure mode. So C_1 = 1, and C_2^n = 0. If we consider the m most probable states, we have:

$$C = \sum_{i=1}^{m} C_i \cdot P_i + \sum_{i=m+1}^{2^n} C_i \cdot P_i \quad . \tag{4}$$

From Eqn. 3, we have:

$$\sum_{i=m+1}^{2^n} C_{2^n} \cdot P_i \le \sum_{i=m+1}^{2^n} C_i \cdot P_i \le \sum_{i=m+1}^{2^n} C_1 \cdot P_i \quad . \tag{5}$$

Since C_1 = 1, and C_2^n = 0, Eqn.5 becomes

$$0 \le \sum_{i=m+1}^{2^n} C_i \cdot P_i \le \sum_{i=m+1}^{2^n} P_i = 1 - \sum_{i=1}^{m} P_i \quad . \tag{6}$$

Substituting Eqn. 6 into Eqn. 4, we get:

$$\sum_{i=0}^{m} C_i \cdot P_i \le C \le \sum_{i=0}^{m} C_i \cdot P_i + 1 - \sum_{i=0}^{m} P_i \quad (7)$$

Table 3 gives the connectivity of the network between Lavazè Pass and Riccomassimo which are two remote places in Trentino Province located at the north and south path of the APT region as shown in Fig. 4. When the 10^2 most probable states are considered, the range becomes [0.7852, 0.8607]. As the value of m increases, the upper and lower bounds of C converge quickly. When m is 10^6, the bound is [0.8601， 0.8606], so we can say that the connectivity between Lavazè Pass and Riccomassimo is 0.86.

Table 3 Connectivity reliability between Lavazèand Riccomassimo in APT network

The value of m		$m = 10^2$	$m = 10^3$	$m = 10^4$	$m = 10^5$	$m = 10^6$
Connectivity	MC	1	1	0.9998	0.9999	0.9999
	ORDER-II	[0.7852, 0.8607]	[0.8463, 0.8606]	[0.8532, 0.8606]	[0.8584, 0.8606]	[0.8601， 0.8606]
Computation time(s)	MC	3.6091	9.0381	71.4162	636.5239	6259
	ORDER-II	3.1162	9.858	83.8680	1229	89029

On the other hand, when we use MC method, the results converge slowly. When m = 10^6, the connectivity is 0.9999. It is much larger than the real value which is around 0.86. As for the computation time, MC method is much faster than ORDER algorithm. When m = 10^6, the computation time for MC method is 6259s, while for ORDER algorithm it is 89029s.

Conclusion

This paper compared the Monte Carlo method and ORDER algorithm in calculating the connectivity reliability in APT network. The comparison shows that ORDER algorithm can give a more accurate result than Monte Carlo method. While Monte Carlo method is faster than ORDER algorithm. The results are very helpful for bridge managers and government officials in understanding the network status.

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