ADER Schemes on Adaptive General Triangulations for the Solution of Scalar Conservation Laws

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Introduction

In general, multi-dimensional problems of the form
\[
\frac{\partial}{\partial t} u + \nabla f(u) = 0
\]
involve complicated geometries in real world applications. Therefore, ADER schemes using Arbitrary high order DERivatives are constructed on adaptive triangulations to achieve high resolution and flexibility.

Time Discretisation

The solution of a Generalised Riemann problem at cell interfaces is approximated by the Taylor series expansion
\[
u(r, 0) \approx u(0, 0) + \sum_{k=1}^{m-1} \frac{r^k}{k!} \frac{\partial^k}{\partial t^k} u(0, 0)
\]
All time derivatives are substituted by space derivatives using the Lax-Wendroff procedure
\[
\frac{\partial^k}{\partial t^k} u = (-\lambda)^k \frac{\partial^k}{\partial x^k} u \quad \text{with} \quad \lambda = \frac{\partial f}{\partial u}
\]
resulting in an explicit, one-step scheme of order \(m\).

Space Discretisation

The domain is discretised by triangular Finite-Volume cells carrying cell average values. Polynomials of degree \(m-1\) are reconstructed on each cell to provide the required space derivatives up to order \(m-1\). Numerical fluxes are computed at cell interfaces to update the cell average values via
\[
\bar{u}_{i}^{n+1} = \bar{u}_{i}^{n} - \frac{T}{|T_i|} \sum_{\sigma=1}^{3} \mathcal{F}_{i,\sigma}^{n}
\]

Stencils for Polynomial Reconstruction

1) Centered stencils can be used to reconstruct polynomials in the Least Square sense. TVD limiters are necessary to avoid oscillations.

2) WENO reconstructions can be used, which require a whole set of stencils. A smoothness indicator provides nonlinear weights. The convex combination of all recovered polynomials gives the final reconstruction polynomial.

Numerical Results

Convergence of a 3rd order ADER scheme with WENO reconstruction for the linear advection equation with smooth initial condition
\[
u_0(x, y) = \sin(2\pi(x + y))
\]

Rotating Slotted Cylinder

Burgers Equation

Acknowledgement

This work is supported by the European Union through the research and training network NetAGES.

References


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