Can turbulence anisotropy suppress horizontal circulation in lakes?

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Abstract

Different types of circulation can develop in closed basins as a result of spatially variable wind forcing, geometrical variations and turbulence anisotropy. In this work, focussing on the latter aspect and considering for simplicity an idealized rectangular basin, we analyse the competition among horizontal and vertical circulation in different conditions (e.g. wind and depth spatial variations). We solve the problem numerically with the widely used Princeton Ocean Model (POM), and filter the wave dynamics to obtain indications about the asymptotic steady state conditions. Thus, we show that different flow fields develop depending on the ratio between the vertical and horizontal eddy viscosity calculated by the model. For instance, the same spatial variation of the wind shear stress can produce different circulations depending on the turbulence anisotropy arising from different wind velocity. Therefore, we claim that a careful estimation of the horizontal eddy viscosity is important, although its role is often underestimated.

1. Introduction

The development of a horizontal circulation (i.e. with a significant net component in terms of depth averaged velocity) in lakes and reservoirs may be due to several factors: an inhomogeneous wind forcing, a variable bathymetry, the effect of Coriolis, etc. On the other hand, the presence of a density stratification is typical of most lakes, especially during summer, when the temperature profile shows stronger vertical gradients. In such conditions, the vertical transmission of momentum, mass and heat due to turbulent eddies is inhibited along the vertical direction, causing a relevant deviation of the eddy viscosity, diffusivity and conductivity coefficients from isotropic values. This means that the diffusion of velocity is favoured along the horizontal direction with respect to the vertical, making the velocity distribution more homogeneous horizontally and letting it vary along the depth, with possible inversion of the direction (i.e. vertical circulation). As a result, also in presence of factors that could determine the onset of a horizontal circulation, the turbulence anisotropy may reduce the lateral variations of velocity. Building on a recently published analytical solution (Toffolon and Rizzi, 2009), where the role of the anisotropic transmission of momentum was investigated together with the main geometrical features in a highly schematized basin, we compute the flow field numerically (hence solving the complete non-linear system of differential equations) to understand whether the predictions of the analytical model are confirmed. This is a further step towards the consideration of the role of turbulence anisotropy in practical real cases, where the horizontal eddy viscosity estimates are usually based on analogies with measurements and results that are available about large scale horizontal mixing (e.g., Okubo, 1971; Peeters et al., 1996; Okely et al, 2010).
In this work, we use the well-known Princeton Ocean Model (POM), one of the oldest and most popular models for lake hydrodynamics, in order to reconstruct a typical simulation framework. The model can simulate the complete unsteady hydrodynamics, but we will filter the wave dynamics to obtain indications about the asymptotic steady state conditions and the residual circulation induced by a long lasting wind field.

2. Formulation of the problem

The Princeton Ocean Model solves the Reynolds averaged equations assuming the hydrostatic distribution of pressure and the Boussinesq approximation (Blumberg and Mellor, 1987). The physical domain is discretised using boundary-following σ-coordinates. The vertical eddy viscosity and diffusivity are calculated by means of the Mellor-Yamada 2.5 level model (Mellor and Yamada, 1974), whereas the horizontal coefficients are estimated according to the standard approach introduced by Smagorinsky (1963).

![Figure 1: (a) Sketch of the simplified rectangular basin with laterally variable depth and wind forcing. (b) Initial temperature profile.](image)

The water body is represented by means of a simplified rectangular shape, with dimensions \(L_x\) and \(L_y\) along the major axis and the transversal direction, respectively, whereas the depth \(D\) is assumed to vary linearly with respect to an average value \(D_0\) with a lateral slope \(s\). A sinusoidal variation along the lateral direction is assumed for the wind shear stress:

\[
\tau = \tau_0 \left[1 + \phi \cos \left(\frac{\pi y}{L_y}\right)\right],
\]

where \(\tau_0\) is the reference shear stress and \(\phi\) is the coefficient of variation. Hereafter, two values of the wind velocity \(W\) are assumed as reference cases: (a) 2 m/s for a weak wind, and (b) 10 m/s for a strong wind. These can be easily transformed into the equivalent wind shear stress through one of the several empirical relationships (e.g. Wüst and Lorke, 2003).

<table>
<thead>
<tr>
<th>Table 1: Geometrical characteristic of the simplified basin.</th>
</tr>
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<tbody>
<tr>
<td>Length (L_x) [m]</td>
</tr>
<tr>
<td>-------------------</td>
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<td>6000</td>
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</table>

Toffolon and Rizzi (2009) proposed a simplified analytical solution for the flow field in the central part of an elongated water body (with a constant depth) and subject to a laterally
variable wind forcing as the one described by equation (1). In this case, only a few parameters are supposed to affect the developing circulation. One is obviously the coefficient of variation of the external forcing $\phi$, while a second parameter has been identified in the ratio

$$\alpha = \frac{D_0^2/v_z^2}{L_y^2/v_y^2} = \frac{D_0}{L_y} \frac{v_y^2}{v_z^2}, \quad (2)$$

where $v_z'$ is the vertical turbulent viscosity and $v_y'$ the horizontal turbulent viscosity. The first equality in (2) suggest that the parameter $\alpha$ represents the ratio between the temporal scales of the diffusion of momentum in the vertical and in the horizontal directions. However, once that the geometrical ratio is fixed (typically $L_y >> D_0$), the parameter essentially depends on the degree of turbulence anisotropy (typically $v_y' >> v_z'$). Having this conceptual framework in mind, the interpretation of the results of the numerical simulations is definitely easier.

In order to give a quantitative measure of the predominance of horizontal or vertical circulation in a given cross-section, Toffolon and Rizzi (2009) also introduced a circulation number in the form

$$\omega = \frac{1}{L_y} \int_0^{L_y} a \, dy, \quad a(y) = \int_0^D |u| \, dz$$

which represents the width-averaged ratio between the depth-averaged residual longitudinal velocity and its possible maximum value (when no inversions occur in the profile). Therefore, small values of $\omega$ identify a vertical circulation, whereas when $\omega$ approaches 1 the circulation tends to develop in the horizontal plane, and hence to be well described by depth-averaged models.

### 3. Results and discussion

The hydrodynamic field and the coefficients of vertical and horizontal eddy viscosity have been obtained with the standard version of POM, in order to reproduce what a typical user can get from the application of a consolidated numerical model. We have run simulations in the simplified basin for three days with a constant wind field, assuming adiabatic conditions for the temperature (i.e., it can only diffuse in the water body without external exchanges) and starting from the profile shown in Figure 1b. The constant width and the adiabatic assumption are not fully realistic, but contribute to simplify the problem and to infer more general conclusions. In particular, the aim is to analyse the steady circulation and let the initial wavy motion decade. Hence, all results are plotted after a time $t = 3$ days, and, in order to filter residual wave activity that is dampening slowly, the plotted variables are time-averaged over the last 2 days. Steady conditions cannot be defined strictly since the temperature is diffusing vertically and the initial stratification is modified irreversibly. Therefore, although arbitrarily defined, the fixed average procedure allows one to represent the results in a coherent way. We consider two wind velocities (weak, $W = 2$ m/s, and strong, $W = 10$ m/s), which produce two different turbulence intensities. While the horizontal eddy viscosity does not change significantly with the water velocity, the stronger wind produces a much higher vertical eddy viscosity (as computed by the Mellor-Yamada model) than the weaker wind. Hence, weak winds are likely to produce highly anisotropic conditions ($v_z' << v_y'$, large $\alpha$), whereas strong winds tends to bring the system towards a more isotropic status (small $\alpha$).
Figure 2: Superficial flow field (time-averaged longitudinal velocity \( u_m \)) for constant depth (\( z=0 \)), variable wind (\( \phi=0.3 \)), and two values of wind velocity \( W \): (a) 2 m/s, (b) 10 m/s.

Figure 3: Flow field (time-averaged longitudinal velocity \( u_m \)) in the cross-section \( x=L_x/2 \) for constant depth (\( z=0 \)), variable wind (\( \phi=0.3 \)), and two values of wind velocity \( W \): (a) 2 m/s, (b) 10 m/s.

As a first test, we examine the case of constant depth (\( z = 0 \)), moderately variable wind (\( \phi = 0.3 \)), neglecting the contribution of Coriolis acceleration (\( f = 0 \)). In Figure 2b it is possible to distinguish a clear horizontal circulation developing for the stronger wind (10 m/s), whereas a vertical circulation tends to prevail for the weak wind (2 m/s, Figure 2a). Focussing the attention on the transversal cross-section in the centre of the basin (\( x = L_x/2 \)) to assess the type of circulation, it is easy to recognize the typical patterns of the flow field (Figure 3). These are prevalently associated with the value of the vertical eddy viscosity: in the case of weaker wind, vertical transfer of momentum is less intense and is significantly smaller than the horizontal transfer, causing a high lateral shear from the region of high wind velocity, which tends to move also the opposite side of the superficial layer.

The role of the turbulence anisotropy in shaping the flow field is analysed by imposing a fictitious amplification/reduction of the horizontal eddy viscosity to approximately match the anisotropy degree characteristic of the two cases in Figure 2, but inverting the values. More specifically, we have multiplied all the horizontal eddy coefficients for the coefficient \( \gamma \) reported in Table 2, where also other quantitative details are provided about the simulations.
The effect of inverting the anisotropy degree are shown in Figures 4 and 5, where the type of circulation (horizontal/vertical) changes significantly. This is evident in Figure 4b for $W = 10$ m/s: the “natural” horizontal circulation of Figure 2b (due to the more isotropic turbulence), changes into an almost vertical circulation, caused by the more pronounced anisotropy (i.e. horizontal transfer of momentum is much larger than the vertical one). The opposite case is not so clear (Figure 4a), likely due to the presence of a large numerical viscosity (possibly larger than the small imposed value $\sim 10^3$ m$^2$/s for $v^2_t$) that precludes the possibility to obtain an actual modification of the anisotropy. For this reason, the expected increase of horizontal circulation due to the more isotropic condition (with respect to Figure 2a) is only marginal. An indirect evidence is discussed at the end of the paper.

Table 2: Numerical simulations: main input and output parameters. Notes: * the values are calculated for the epilimnion; ** the value imposed to the eddy viscosity coefficient may be smaller than the actual numerical viscosity, so the actual value of $\alpha$ acting in the numerical simulations may be underestimated. The output parameters for the cases with lateral slope (6a, 6b) are estimated with respect to inclined $\sigma$-grid layers and may not be accurate. The last column of the table reports the qualitatively assessed circulation type: $v$ for vertical, $h$ for horizontal circulation.

<table>
<thead>
<tr>
<th>id figure</th>
<th>$W$ [m/s]</th>
<th>$s$ [-]</th>
<th>$\phi$ [m/s]</th>
<th>$f$ [s$^{-1}$]</th>
<th>horiz. turb. alteration $\gamma$</th>
<th>$\nu_{z, epi}$ [m$/s$]</th>
<th>$\nu_{z, epi}$ [m$/s$]</th>
<th>$\alpha^*$ [-]</th>
<th>$\omega^*$ [-]</th>
<th>circ. type</th>
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<td>8.3 · 10$^{-2}$</td>
<td>2.4 · 10$^{-4}$</td>
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<td>0.59</td>
<td>v</td>
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<tr>
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<td>0.3</td>
<td>0</td>
<td>1</td>
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<td>3.2 · 10$^{-2}$</td>
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<td>0.89</td>
<td>h</td>
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<td>0</td>
<td>0.02</td>
<td>2.3 · 10$^{-3}$**</td>
<td>2.1 · 10$^{-4}$</td>
<td>0.13**</td>
<td>0.68</td>
<td>v</td>
</tr>
<tr>
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<td>0.3</td>
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<td>v</td>
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<tr>
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<td>3.0 · 10$^{-2}$</td>
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<td>3.4 · 10$^{-2}$</td>
<td>0.45</td>
<td>0.63</td>
<td>v</td>
</tr>
</tbody>
</table>

Figure 4: Superficial flow field as in Figure 2, but with modified turbulence anisotropy, for: (a) wind velocity $W = 2$ m/s with reduced horizontal eddy viscosity (1/50) to reproduce the epilimnetic anisotropy degree associated with strong wind (10 m/s, Figure 2b); (b) wind velocity $W = 10$ m/s with increased horizontal eddy viscosity (50x) to reproduce the epilimnetic anisotropy degree associated with weak wind (2 m/s, Figure 2a).
Further analyses on the effect of the turbulence anisotropy (due to different wind velocities) on the development of horizontal vs. vertical circulation can be illustrated considering other factors that tend to make the flow field deviate from the symmetrical condition of pure vertical circulation (described, e.g., by Heaps, 1984): depth variations and Coriolis force. The effect of a lateral slope of the lake bottom is shown in Figure 6 for the two wind velocities, suggesting that a horizontal circulation develops for the stronger wind and more isotropic conditions (Figure 6b), while a more complex, yet prevalently vertical, motion results for the weak wind (Figure 6a). The effect of including the Coriolis force is illustrated in the two plots of Figure 7, which confirms the same qualitative behaviour as the previous cases.

Finally, we re-examine two of the previous cases looking for evidences of how changing the anisotropy can change the nature of the circulation. In Figure 8a we use the same conditions as in Figure 3a, where a vertical circulation developed due to the weak anisotropy, but we imposed constant eddy coefficients in the epi-, meta- and ipolimnion, with the values...
estimated in the case of Figure 3b (strong wind, almost isotropic turbulence, horizontal circulation). In this artificial case a clear horizontal motion develops, although characterized by a small scale of velocity because of the unrealistically large vertical eddy viscosity for the weak imposed wind (more details about the typical scale of velocity are given, e.g., in Toffolon and Rizzi, 2009). Unlike the case discussed in Figure 5a, turbulent viscosity is characterized by higher values (much higher than numerical viscosity) and the resulting circulation is predominately horizontal, as one would expect. On the contrary, the case in Figure 8b shows how an artificial increase (50x) of the anisotropy tends to modify the horizontal motion of Figure 7b in a predominantly vertical circulation.

Figure 7: The effect of the Coriolis force \( f = 10^{-4} \text{s}^{-1} \), in the case of constant depth and wind \( s=0, \phi=0 \), for two values of the wind velocity \( W \): (a) 2 m/s, (b) 10 m/s.

Figure 8: Two separate cases showing the effect of imposed anisotropy: (a) the same case as in Figure 3a with \( W = 2 \) m/s, but with constant coefficients (three layers) obtained from the case of Figure 3b; (b) the same case as in Figure 7b with \( W = 10 \) m/s, but with increased horizontal eddy viscosity (50x).

Table 2 summarizes the main parameters describing the results, together with a qualitative assessment of the type of circulation. Recalling that high \( \omega \) means horizontal circulation (a threshold \( \omega >0.7 \) can be approximately derived from a comparison with the qualitatively
assessment, although with a few exceptions), and that this is due to low anisotropy (i.e. small $\alpha$), it clearly emerges that all the cases with horizontal circulation are denoted by low values of $\alpha$ (in the range 0.14-0.15 for these runs). The only exception is the case of Figure 5a, which however is likely influenced by numerical diffusion. On the contrary, vertical circulation (low $\omega$) develops for larger $\alpha$ (>0.24 for these runs). This confirms that $\alpha$, together with the other parameters describing the external forcing and geometrical variations, is a significant parameter for assessing the nature of the circulation.

4. Conclusions

We have shown that different flow fields develop depending on the ratios between the typical geometrical scales and between the vertical and horizontal viscosity coefficients. In order to individuate the characteristic values of the anisotropic eddy viscosity in response to external wind forcing, a popular numerical model (POM) has been used. We have noted that the same spatial variation of the wind shear stress can produce different circulations depending on the turbulence anisotropy that arise because of different velocities of the wind. Unlike one would expect, it is not the speed of wind itself that determines the type of circulation, but the corresponding turbulence anisotropy degree. In fact, maintaining the same wind forcing but inverting the values of turbulent viscosity, the resulting circulation is the opposite one. Other factors have been analysed that can modify the type of circulation (namely depth variation and Coriolis acceleration) obtaining the same qualitative behaviour as for the previous case. In the light of the presented results, it is important to properly evaluate the turbulent anisotropy degree. Therefore, we claim that a careful estimation of the horizontal eddy viscosity is important, although its role is often underestimated.

References


