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# Evolutionary fractal theory of erosion and experimental assessment on MIR space station

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**Abstract**

In this paper, we present a new approach to study erosion phenomena. It is substantially based on multi-scale and fractal concepts. The theory permits to predict the wear in terms of mass loss evolution for one-, two- and three-dimensional structures under erosive flows. In particular, a comparison with the results of an experimental investigation—only recently published—performed during 28 and 42 months of in-flight exposure for the MIR orbital space station, is presented. The experimental analysis gives the mass loss of polymer films, used as protection against erosion due to space debris impacts, as a function of in-flight exposure time, as well as their life-time predictions, crucial parameters for an optimal design of the protective films. The theoretical approach predicts a catastrophic damage evolution in terms of eroded mass as a function of the exposure time. This surprising time-effect is experimentally confirmed, showing that the trivial assumption of steady-state damage evolution is strongly non-conservative.


*Keywords:* Erosion; Fractal; Non-linear damage-evolution; Space debris; Life-time

## 1. Introduction

Classical theory of wear, proposed by Reye [1] about 140 years ago and universally accepted, assumes a removed volume proportional to the energy dissipated in the process. On the other hand, the universal fractal laws only recently published by Carpinteri and Pugno for the evaluation of the energy dissipation during fragmentation and comminution [2], show a possible violation of the Reye assumption. Based on statistical and fractal concepts, the three-dimensional law predicts an energy dissipation proportional to the measure of a fractal domain always comprised between a surface and a volume. As a consequence, the Reye assumption appears only as a limit case.

The wear phenomenon is studied in the tribology science embracing the scientific investigation of all types of friction, lubrication and wear [3].

Focusing our attention on wear, we can distinguish main forms like adhesive, abrasive, corrosive, surface fatigue (occurring during repeated sliding or rolling over a track), fretting (occurring when contacting surfaces undergo oscillatory tangential displacement of small amplitude), impact, brittle fracture and erosion wear mechanisms. In particular erosion, in which a particle carried in a fluid medium hits a solid surface and removes material from it, can involve low-speed, high-speed as well as cavitation. High-speed erosion is absolutely the predominant wear process in aerospace environment, due to space debris impacts [4–13]. In fact, orbital debris generally moves at very high-speeds. In low earth orbits (altitudes lower than 2000 km) the average relative velocity at impact could be around 10 km/s. At this velocity, even small particles contain a significant amount of kinetic energy. For example, a metal particle having size around 1 mm has damage potential similar to that of a 22 caliber long rifle bullet. Fragments, typically smaller than 1 mm in size, do not generally pose hazard to spacecraft functionality but they strongly erode the aerospace vehicles.

In this paper, we present a new approach to study the erosion phenomena. It is substantially based on the above mentioned universal fractal laws [2]. The theory makes possible the prediction of the time history of mass loss under erosion and the life-time of the elements. In addition, a comparison between the theory and the results of an experimental investigation—only recently published [14]—performed during 28 and 42 months of in-flight exposure for the MIR

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orbital space station, is presented. The life-times of the protective films, crucial parameters for the space-station safety, have been predicted according to our theory, showing a methodology towards their optimal design. The trivial assumption of steady-state damage evolution is demonstrated to be strongly non-conservative.

## 2. Classical erosion

The wear loss $w_1$ during erosion is defined as the ratio of the removed mass $M_1$ to the incident mass $M_2$ causing erosion [15,16]:

$$w_1 = \frac{\mathrm{d}M_1}{\mathrm{d}M_2} \tag{1}$$

On the other hand, the wear coefficient $k_1$ due to erosion is defined as

$$w_1 = k_1 \frac{\rho_2 v_2^2/2}{H_1} \tag{2}$$

where $\rho_2$ is the density of the incident particles of (mean square) velocity $v_2$ and $H_1$ is the hardness of the eroded material. It is very interesting to note that the ratio between $k_1$ and $w_1$ (with $H_1 = F/A$ and $F$ and $A$, respectively, contact force and area) is equivalent to the well-known drag coefficient $C_x = F/(1/2\rho v^2 A)$ in hydrodynamics.

The main difference between the wear loss $w_1$ and the wear coefficient $k_1$ during erosion is that the former is not a material property, being velocity-dependent. On the other hand, the latter, in the classical approach, can be considered as a material property and is obviously velocity-independent.

Since $M_2$ is defined as the incident mass causing erosion, the energy dissipated in the process will be

$$\mathrm{d}W_1 = \tfrac{1}{2} v_2^2 \,\mathrm{d}M_2 \tag{3}$$

where $W_1$ is the work done to erode the target material (1) and the rebound energy, a fraction of the total incident kinetic energy (material 2) is excluded by definition. From Eqs. (2) and (3), we obtain

$$w_1 = \frac{k_1 \rho_2}{H_1} \frac{\mathrm{d}W_1}{\mathrm{d}M_2} \tag{4}$$

If we define the wear resistance as $S_1 = W_1/V_1$, ratio of dissipated energy to removed volume, from Eq. (1) we have

$$w_1 = \frac{k_1 \rho_2}{H_1} \frac{\mathrm{d}W_1}{\mathrm{d}M_2} = \frac{k_1 \rho_2}{H_1 \rho_1} \frac{S_1 \,\mathrm{d}M_1}{\mathrm{d}M_2} = \frac{k_1 \rho_2}{H_1 \rho_1} S_1 w_1 \tag{5}$$

where $\rho_1$ is the density of the target material.

Eliminating $w_1$ from Eq. (5), we obtain the wear resistance $S_1$, which is a macroscopic parameter, as a function of microscopic material constants:

$$S_1 = \frac{H_1 \rho_1}{k_1 \rho_2} \tag{6}$$

## 3. Classical coupled erosion

The developed theory has described the relationship between the classical wear coefficient $k_1$ and the wear resistance $S_1$ (of the material with hardness $H_1$). It is interesting to emphasize that relationship (6) is true only for a pure erosion process, i.e. when we assume that the whole energy is entirely dissipated in erosion. This cannot be assumed (by definition) in a coupled theory, for which the definition (6) must be modified taking into account that not the whole energy is dissipated to erode only one of the two materials. According to these considerations, the hypothesis used in Eq. (3) must be replaced with the following relationships:

$$\mathrm{d}W_i = \alpha_i \,\mathrm{d}W, \quad \alpha_i = \frac{W_i}{W} \tag{7a}$$

$$\sum_i \alpha_i = 1 \tag{7b}$$

where $\alpha_i$ is the ratio of the energy $W_i$, dissipated to erode the first material ($i = 1$) or the second one ($i = 2$), to the total energy involved in the process. As a consequence, Eq. (6) becomes

$$S_i = \alpha_i \frac{H_i \rho_i}{k_i \rho_j} \tag{8}$$

where $H_i$ and $k_i$ being the hardness and the wear coefficient during erosion for the first ($i = 1$) or second ($i = 2$) materials.

Eq. (8) permits to obtain theoretically the ratio between the volumes removed from the two different materials (coupled parameter):

$$\frac{\alpha_2}{\alpha_1} \equiv \frac{W_2}{W_1} = \frac{S_2 V_2}{S_1 V_1} = \frac{\alpha_2 \rho_2 H_2}{\rho_1 k_2} \frac{\rho_2 k_1}{\alpha_1 \rho_1 H_1} \frac{V_2}{V_1} \tag{9}$$

from which the coupled law becomes

$$\frac{V_2}{V_1} = \left(\frac{\rho_1}{\rho_2}\right)^2 \frac{k_2 H_1}{k_1 H_2} \tag{10}$$

and predicts a linear relationship between the eroded volumes for the two materials, as a function of other parameters like density, hardness and wear erosion coefficient.

## 4. Fractal erosion

In this section a multi-scale and fractal theory, extending the classical concepts developed previously, is presented.

The fundamental hypothesis of the theory is a self-similar distribution of the energy dissipations. It is realistic for different erosion mechanisms like brittle (self-similar fragments at each scale) as well as ductile (self-similar ploughs) erosions. Note that that theory is self-consistent, the space debris, deriving from a previous fragmentations (e.g. of asteroids), being expected to be self-similar in size and correspondingly to cause a self-similar damage in the target.

The statistical theory is substantially based on the fractal universal laws for energy dissipation during fragmentation [2] for one-, two-, and three-dimensional objects, that, concerning the mean values, can be summarized as follows:

$$W = \Gamma_1^* L^{\gamma_1}, \quad 0 \leq \gamma_1 \leq 1 \tag{11a}$$

$$W = \Gamma_2^* A^{\gamma_2}, \quad \tfrac{1}{2} \leq \gamma_2 \leq 1 \tag{11b}$$

$$W = \Gamma_3^* V^{\gamma_3}, \quad \tfrac{2}{3} \leq \gamma_3 \leq 1 \tag{11c}$$

i.e. the energy $W$ dissipated in the comminution processes is proportional to the length $L$/surface $A$/volume $V$ of the element, raised to fractal exponents $\gamma_i$, for which well defined limits are given. $\Gamma_i^*$ is the so-called *fractal fragmentation strength* and appears to be a constant, differently from the usual fragmentation strength $S$ [17]. Strong size effects on classical fragmentation strength $S$ are in fact clearly observed from an experimental viewpoint [17,18].

The statistical three-dimensional law of Eq. (11c) can be considered as the generalization of the Reye's hypothesis [1]. As a matter of fact, the energy dissipation is classically assumed as occurring in a volume, for which $\gamma_3 = 1$. On the other hand, if the dissipation occurs on a surface, the fractal exponent becomes $\gamma_3 = 2/3$. In general, it occurs over a fractal domain comprised between a surface and a volume, so that $2/3 \leq \gamma_3 \leq 1$. It is interesting to emphasize that the physical dimensions of $\Gamma_3^*$ changes with $\gamma_3$ and that they become those of a pressure only in the classical case of $\gamma_3 = 1$. Similar considerations are related to the one- and two-dimensional cases.

For three-dimensional target, we can define a fractal wear loss $w_1^*$ due to erosion, generalizing the classical concept of $w_1 \equiv w_{3,1} = \rho_1 V_1/\rho_2 V_2$ (we assume the convention for which in the first pedex ($i = 1, 2, 3$) corresponds to the space-dimension of the object, as well as the second one ($j = 1, 2$) defines the material) as proportional to the energy dissipated in the erosion wear process:

$$w_{3,1}^* = \frac{\rho_1 V_1^{\gamma_{3,1}}}{\rho_2 V_2} \tag{12}$$

On the other hand, the fractal wear coefficient $k_{3,1}^*$ due to erosion, can be defined as

$$w_{3,1}^* = k_{3,1}^* \frac{\rho_2 v_2^2/2}{H_1} \tag{13}$$

The main difference between the fractal wear loss $w_{3,1}^*$ and the fractal wear coefficient $k_{3,1}^*$ is that the former is not a material property, being velocity-dependent. On the other hand, $k_{3,1}^*$ can be considered as a real material constant with anomalous physical dimensions changing with $\gamma_{3,1}$. It is interesting to emphasize that only in the classical case ($\gamma_{3,1} = 1$) it is a dimensionless parameter.

The energy dissipated during a pure erosion wear process can be obtained from Eq. (3):

$$w_{3,1}^* = \frac{\rho_1 V_1^{\gamma_{3,1}}}{\rho_2 V_2} = \frac{k_{3,1}^* \rho_2}{H_1} \frac{\mathrm{d}W_1}{\mathrm{d}M_2} \tag{14}$$

From Eq. (14) and from the definition of fractal wear strength (see Eq. (11c)) we have

$$w_{3,1}^* = \frac{k_{3,1}^* \rho_2}{H_1} \frac{\Gamma_{3,1}^* V_1^{\gamma_{3,1}}}{M_2} = \frac{k_{3,1}^* \rho_2}{H_1 \rho_1} \Gamma_{3,1}^* w_1^* \tag{15}$$

Eliminating $w_{3,1}^*$ from Eq. (15), we obtain the fractal wear resistance $\Gamma_{3,1}^*$ as

$$\Gamma_{3,1}^* = \frac{H_1 \rho_1}{k_{3,1}^* \rho_2} \tag{16}$$

The developed theory has established the relationship between the fractal wear coefficient $k_{3,1}^*$ and the fractal wear resistance $\Gamma_{3,1}^*$ (of the material with hardness $H_1$) due to erosion. For $\gamma_{3,1} = 1$, Eq. (16) becomes Eq. (6) (energy dissipation assumed to occur in a volume). $\Gamma_{3,1}^*$ is a macroscopic parameter and has been previously obtained as a function of microscopic material constants, like the hardness $H_1$ and the fractal wear coefficient $k_{3,1}^*$ of the base material.

Generalizing Eq. (16) for the other space-dimensions, we obtain by definition:

$$\Gamma_{i,1}^* = \frac{H_1 \rho_1}{k_{i,1}^* \rho_2} \tag{17}$$

## 5. Fractal coupled erosion

It is important to emphasize that Eq. (17) is true only for a pure erosion process, i.e. we have assumed that the whole power is entirely dissipated to erode only one of the two materials. This cannot be assumed (by definition) in a coupled theory, for which the definition (17) must be modified taking into account that not the whole energy is dissipated in wearing the same material. According to these considerations, Eq. (3) must be replaced with Eq. (7) and, consequently, Eq. (17) becomes

$$\Gamma_{i,1}^* = \alpha_1 \frac{H_1 \rho_1}{k_{i,1}^* \rho_2}, \quad 1 \leftrightarrow 2 \tag{18}$$

Eq. (18) permits to obtain theoretically the relationship between the volume removals during coupled wear processes. Assuming, for example, three-dimensional impacting particles against two-dimensional objects:

$$\frac{\alpha_2}{\alpha_1} \equiv \frac{W_2}{W_1} = \frac{\Gamma_{3,2}^* V_2^{\gamma_{3,2}}}{\Gamma_{2,1}^* A_1^{\gamma_{2,1}}} = \frac{\alpha_2 \rho_2 H_2}{k_{3,2}^* \rho_1} \frac{k_{2,1}^* \rho_2}{\alpha_1 \rho_1 H_1} \frac{V_2^{\gamma_{3,2}}}{A_1^{\gamma_{2,1}}} \tag{19}$$

from which the fractal coupled law becomes

$$\frac{V_2^{\gamma_{3,2}}}{A_1^{\gamma_{2,1}}} = \left(\frac{\rho_1}{\rho_2}\right)^2 \frac{H_1 k_{3,2}^*}{H_2 k_{2,1}^*} \tag{20}$$

Accordingly, we can predict the eroded surface area as a function of the impacting volume:

$$A_1 \propto V_2^{\gamma_{3,2}/\gamma_{2,1}} \tag{21}$$

## 6. Experimental comparison with in-flight erosion due to space debris impacts on MIR orbital space station

The results of an experimental investigation performed during 28 months (17 July 1995 to 12 November 1997) and 42 months (17 July 1995 to 8 January 1999) of in-flight exposure of MIR orbital space station, have been only recently published [14]. In particular, the experimental analysis has permitted to obtain the mass loss of polymer films used as protection against erosion due to space debris impacts, as a function of in-flight exposure time. The aim of this section is a comparison between the fractal theory of erosion and the mentioned experimental data. For experimental details the reader should refer to [14].

The energy available for the erosion of the films is the fraction $(1-\zeta)\alpha_1$ of the total kinetic energy of the impacting particles, $\zeta$ being the fraction of rebound energy. The kinetic energy available is proportional to the cumulative mass $M_2$ of space debris impacting against the MIR space station. Considering a steady-state flow of space debris (confirmed by the same damage evolutions observed in the different oriented protective films), i.e. $M_2 \propto t$, where $t$ is the time of exposure and observing that, if $L_1$ denotes a characteristic length of the damage zone having surface area $A_1$, $A_1 \propto L_1^2 \propto (L_1^3)^{2/3} \propto V_1^{2/3} \propto M_1^{2/3}$, where $M \equiv M_1$ is the film mass loss, from Eq. (21) we obtain

$$M \propto t^{\beta}, \quad 1 \leq \beta \leq 3 \tag{22a}$$

The constant of proportionality in Eq. (22a) has a clear physical meaning, so that it can be rewritten as

$$\frac{M}{M_u} = \left(\frac{t}{t_u}\right)^{\beta}, \quad 1 \leq \beta \leq 3 \tag{22b}$$

where $M_u$ is the initial film mass (a variation of which defines the origin of the time) and $t_u$ is the film's life-time. This surprising result is a catastrophic statistical prediction for the damage evolution as a non-linear function of the exposure time. The fractal exponent for the exposure time is theoretically expected to be comprised between 1 and 3 and not, as more intuitive, close to the unity, that would describe a steady-state damage evolution. This steady-state behavior is obtained only as a limit case.

The experimental results in terms of mass loss versus exposure time for the different polymer films, as well as the experimental values of the fractal exponents $\beta$ obtained by a regressive analysis, are reported in Figs. 1–4. As expected, they appear larger than 1, describing a catastrophic damage evolution rather than a steady-state coupled erosive phenomenon. The experimental data for the fractal exponent $\beta$ appear comprised between 1.48 and 3.15, close to the range predicted by the fractal approach within the theoretical limit values of 1 and 3.

As a consequence of the limited number of available data, the corresponding correlation coefficients were found by definition equal to 1. In this context we have to mention that, a stronger assessment of our fractal theory has been successfully found in a similar context, performing an extensive theoretical and experimental analysis on the coupled problem of drilling and wear in mechanical tools [19].

It is interesting to note that, if we remove the hypothesis of coupled processes, i.e. assuming that the entire energy is dissipated in eroding the films, the result formally can be ob-

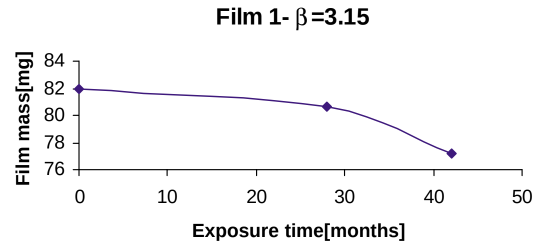


Fig. 1. Experimental mass variation due to space debris erosion during in-flight exposure of MIR space station (Fluoroplast FEP-100A film).

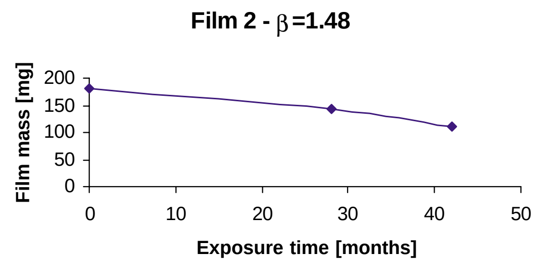


Fig. 2. Experimental mass variation due to space debris erosion during in-flight exposure of MIR space station (Polyimide ΠM-1 ∋ film).

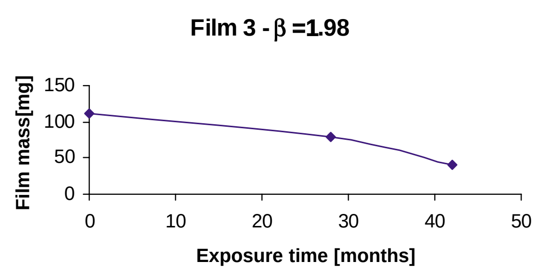


Fig. 3. Experimental mass variation due to space debris erosion during in-flight exposure of MIR space station (Polyimide Kapton 100HN film).

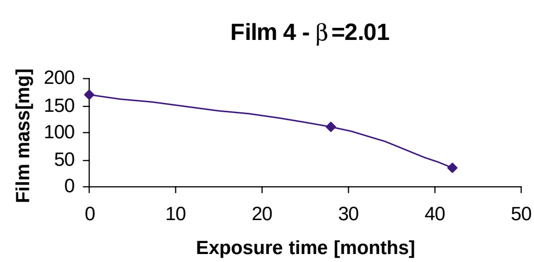


Fig. 4. Experimental mass variation due to space debris erosion during in-flight exposure of MIR space station (aluminized Polyimide ΠM-Y ∋ -OA film).

Table 1
Life time predictions according to our evolutionary fractal theory and to the trivial assumption of a steady-state process for protective polymer films against space debris erosion, during in-flight exposure of MIR space station

| Life time predictions (months) | Fluoroplast FEP-100A | Polyimide ΠM-1 ∋ | Polyimide Kapton 100HN | Aluminized Polyimide ΠM-Y ∋ -OA |
|---|---|---|---|---|
| Evolutionary fractal theory (non-linear) | 103.7 | 80.4 | 53.4 | 47.0 |
| Steady-state assumption (linear) | 710.1 | 110.0 | 65.9 | 52.6 |

tained from Eq. (21) in which $\gamma_{3,2} \equiv 1$. The corresponding domain for the parameter $\beta$ becomes $3/2 \leq \beta \leq 3$, in which substantially the experimental results have been found. This seems to suggest a negligible damage in the harder impacting particles compared to the damage of the softer protective films.

In Table 1, the corresponding life-time predictions, according to our evolutionary fractal theory (obtained by the regressive analysis), are reported and compared with the non-conservative trivial assumption of a steady-state damage evolution.

## 7. Conclusions

The proposed theory is able to predict the damage evolution and the life-time for space structures under multi-scale coupled erosion.

In particular, a comparison with the results of an experimental investigation performed during 28 and 42 months of in-flight exposure of MIR orbital space station [14], has been presented. The experimental and theoretical analyses agree satisfactorily in predicting the mass loss for polymer films used as protection against erosion due to space debris impacts as a function of time during in-flight exposure.

The theoretically expected catastrophic damage evolution for the eroded mass, as a function of the exposure time, clearly emerges from the experimental results. Fractal theory and experiments seem to agree satisfactorily. It is important to emphasize that assuming a trivial steady-state damage evolution in the design of the protective panels, i.e. $\beta = 1$, would strongly reduce the safety of the space station.

In addition, the estimations of the panel life-times are quantified, according to our catastrophic non-linear multi-scale coupled theory and compared with the trivial assumption of a steady-state damage evolution; this would result in not realistic time-effect predictions with dangerous overestimations of the life-times of the protective elements. Accordingly, the trivial assumption of steady-state damage evolution is demonstrated to be strongly non-conservative.

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## References

[1] T. Reye, Zur Theorie der Zapfenreibung, Der Civilingenieur 4 (1860) 235–255.
[2] A. Carpinteri, N. Pugno, One-, two- and three-dimensional universal laws for fragmentation due to impact and explosion, J. Appl. Mech. 69 (2002) 854–856.
[3] E. Rabinowicz, Friction and Wear of Materials, Wiley, New York, 1995.
[4] G. Drolshagen, W.C. Carey, J.A.M. McDonnell, T.J. Stevenson, J.C. Mandeville, L. Berthoud, HST solar array impact survey: revised damage laws and residue analysis, Adv. Space Res. 19 (1997) 239–251.
[5] A.D. Griffiths, J.A.M. McDonnell, G. Drolshagen, Debris production from solar array surface impact spallation: results from the Hubble space telescope, Adv. Space Res. 19 (1997) 253–256.
[6] G. Drolshagen, J.A.M. McDonnell, T.J. Stevenson, S. Deshpande, L. Kay, W.G. Tanner, J.C. Mandeville, W.C. Carey, C.R. Maag, A.D. Griffiths, N.G. Shrine, R. Aceti, Optical survey of micrometeoroid and space debris impact features on EURECA, Planet. Space Sci. 44 (1996) 317–340.
[7] J.A.M. McDonnell, G. Drolshagen, D.J. Gardner, R. Aceti, I. Collier, EURECA's exposure in the near earth space environment. Hypervelocity impact cratering distributions at a time of space debris growth, Adv. Space Res. 16 (1995) 73–83.
[8] R. Aceti, G. Drolshagen, EURECA post flight technology investigation achievements, Acta Astronautica 37 (1995) 347–360.
[9] G. Drolshagen, J.A.M. McDonnell, T. Stevenson, R. Aceti, L. Gerlach, Post-flight measurements of meteoroid/debris impact features on EURECA and the Hubble solar array, Adv. Space Res. 16 (1995) 85–89.
[10] S.Y. Su, D.J. Kessler, Contribution of explosion and future collision fragments to the orbital debris environment, Adv. Space Res. 5 (1985) 25–34.
[11] R.C. Reynolds, Review of current activities to model and measure the orbital debris environment in low-earth-orbit, Adv. Space Res. 10 (1990) 359–372.
[12] S.Y. Su, The velocity distribution of the collisional fragments and its effect on the future space debris environment, Adv. Space Res. 10 (1990) 389–392.
[13] A. Carpinteri, N. Pugno, On the multi-scale damage in aerospace vehicles due to space debris impacts, in: Proceedings of the Italian Group of Fracture Meeting on Damage and Fracture of Materials in Aerospace Environment, Rome, Italy, 26–27 June 2001, pp. 39–43.
[14] V.K Milinchuk, I.P. Shelukhov, T.N. Smirnova, Degradation of polymeric materials under simultaneous action of the space environment factors, in: Proceedings of the Italian Group of Fracture Meeting on Damage and Fracture of Materials in Aerospace Environment, Rome, Italy, 26–27 June 2001, pp. 1–38.
[15] E. Hornbogen, in: K. Friedrich (Ed.), Friction and Wear of Polymer Composites, Composite Materials Series 1, Elsevier, Amsterdam, 1986.

[16] K.-H. Zum-Gahr, Microstructure and Wear of Materials, Tribology Series 10, Elsevier, Amsterdam, 1987.
[17] A. Carpinteri, N. Pugno, A fractal comminution approach to evaluate the drilling energy dissipation, Int. J. Numer. Anal. Methods Geomech. 26 (2002) 499–513.
[18] A. Carpinteri, N. Pugno, A multifractal comminution approach for drilling scaling laws, Powder Technol. 131 (2003) 93–98.
[19] A. Carpinteri, N. Pugno, Fractal coupled theory of drilling and wear, submitted for publication.