Smart Invisible/Visible Synthetic Spider Silk

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Abstract

Spiders suggest to us that producing high strength over density ratio invisible cables could be of great importance. In this paper we show that such invisible cables could in principle be built, thanks to carbon nanotube bundles. We demonstrate that such cables can be easily transported in their visible state (with bunched nanotubes) and that an efficient anti-bunching controllable mechanism can smartly control the visible-in invisible transition, and viceversa. The paper is a synthesis of the analysis reported in [1].

1 Introduction

Carbon nanotubes display unique and extraordinary mechanical properties, such as an extremely high Young’s modulus (~1TPa), strength (~100GPa) and consequently failure strain (~0.1), similar to those of graphite in-plane. Furthermore, the low carbon density (~1300 Kg/m$^3$) suggests that carbon nanotubes have promising high strength and lightweight material applications. In this paper we show that their mechanical properties are sufficient to realize macroscopic invisible cables. The paper is a synthesis of the analysis reported in [1] (please refer to this paper for details; see also the related story “With no visible means of support”, New Scientist, 19 July 2008, 23).

2 Smart invisible/visible synthetic spider silks based on nanotube bundles

Consider carbon nanotubes arranged in a regular lattice with area fraction $\varphi$. The strength $\sigma_c$ of the invisible cable, defined as the failure tensile force divided by the nominal area, is imposed by the equilibrium of the forces to be:
\[ \sigma_c = \varphi \sigma_{NT}, \quad \sigma \rightarrow E, \rho \]  

where \( \sigma_{NT} \) denotes the strength of the single carbon nanotube. The same relation is derived for the cable Young's modulus \( E_c \), considering in eq. (1) the substitution \( \sigma \rightarrow E \) and \( E_{NT} \) as the Young's modulus of the single carbon nanotube, as imposed by the compatibility of the displacements. Similarly, the cable density \( \rho_c \) defined as the cable weight divided by the nominal volume, is predicted according to eq. (1) with the substitution \( \sigma \rightarrow \rho \), where \( \rho_{NT} \) would denote the carbon (nanotube) density, as can be easily derived by the mass balance. Thus, the same (failure) strain \( \varepsilon_c = \sigma_c/E_c = \sigma_{NT}/E_{NT} \) and strength over density ratio \( R = \sigma_c/\rho_c = \sigma_{NT}/\rho_{NT} \) is expected for the cable and for the single nanotube. Eq. (1) can be considered as the simplest law to connect the nanoscale properties of the single nanotube with the macroscopic properties of the cable.

Assuming that the nanotubes are distributed in a regular lattice pattern, one can derive their separation \( p \), from their external diameter \( d_+ \) (internal diameter \( d_- \approx 0 \)) and area fraction \( \varphi \), according to:

\[ p = \left( \sqrt{\frac{\varphi_{max}}{\varphi}} - 1 \right) \frac{d_+}{2} \]  

where \( \varphi_{max} \) stands for the maximum area fraction of a given lattice: \( \varphi_{max} = \pi/(2\sqrt{3}), \pi/4, \pi/(3\sqrt{3}) \) respectively for triangular, square or hexagonal lattices. On the other hand, indicating with \( \lambda \) the light wavelength, the condition for a nanotube to be invisible is:

\[ d_+ \ll \lambda \]  

(3a)

whereas to have a globally invisible cable, we require to not have interference between single nanotubes, i.e.:

\[ p \gg \lambda \]  

(3b)

We do not consider here the less strict limitations imposed by the sensitivity of the human eye, that can distinguish two different objects only if their angular distance is larger than \( \sim 1^\circ \). In other words, we want the cable to be intrinsically invisible.

Assuming \( d_+ / \lambda = 1/10 \), \( p/\lambda \approx 10 \), from the previously reported nanotube theoretical strength, Young's modulus and density, we derive the following wavelength-independent invisible cable properties:

\[ \sigma_c^{(theo)} \approx 10 \text{ MPa}, \quad E_c \approx 0.1 \text{ GPa}, \quad \rho_c \approx 0.1 \text{ Kg/m}^3 \]  

(4)

Thus, with a sufficiently large spacing \( p \), transparent and even invisible cables could in principle be realized. But this very restrictive condition (3b), corresponding to
non interacting nanotubes, evidently implies a low nominal strength. Nevertheless we may note that the condition is sufficient but not necessary. In fact, if eq. (3b) is not verified, e.g. for \( p < \lambda \) thus for interacting nanotubes, we can treat the cable as an aerosol. In this case we can still have a globally transparent cable requiring that its effective refractive index \( n_c \approx 1 + (n_{NT} - 1)\varphi \), \( n_{NT} \) is that of carbon nanotubes, be sufficiently close to the unity, i.e. \( p^2 \gg d_+^2 \), as well as that its effective absorption index \( k_c \approx k_{NT} \varphi \), \( k_{NT} \) is that of carbon nanotubes, multiplied by the cable thickness \( T \) be sufficiently close to zero, i.e. \( T \ll \frac{p^2}{d_+^2 k_{NT}^{-1}} \). Consequently, for this case of interacting nanotubes, the nominal strength is improved but only sufficiently thin sheets can be considered.

The nanotubes in the bundle will tend to bunch due to van der Waals surface attraction. The equilibrium contact width \( w \) of two identical nanotubes with diameter \( d_+ \) (do not subject to forces or constraints and with \( d_- \approx 0 \)) can be determined using contact mechanics as:

\[
    w = 4 \left( \frac{d_+^2 \gamma_s (1 - \nu_{NT})}{\pi E_{NT}} \right)^{1/3}
\]

where \( \nu_{NT} \) is the nanotube Poisson ratio and \( \gamma_s \) is the surface energy. Due to deformation near the contact region of size \( wx \), there is an accompanying stored elastic energy \( \Phi_s \) in the portions of nanotubes in contact (of length \( x \)) that must satisfy the energy balance \( d\Phi_s = 2\gamma_s x dw \); thus, by integration, \( \Phi_s(x) = \frac{\pi E_{NT} d_+^2 x}{128 (1 - \nu_{NT}^2)} \left( \frac{w}{d_+} \right)^4 \). Let us consider the mechanism shown in Figure 1a, in which a moveable platform is introduced along the cable. The total potential energy of the system is

\[
    \Pi(x) = \Phi_s(x) + \frac{\sigma^2}{E_{NT}} AL - (F - F_f)(L - x) + \text{const},
\]

where \( \sigma \) is the resulting tension in the nanotube, having cross-sectional area \( A \) and length \( L \); \( F \) denotes the anti-bunching force applied at the moveable platform and \( F_f \) is the friction force (gravity is here neglected but could be easily included in the energy balance). The energy balance implies \( \frac{\Delta \Pi(x)}{\Delta x} = 2\gamma_s w \), where \( \Delta x \) is a minimum delamination advancement, from which we deduce the simple relation \( F = \frac{3}{2} \gamma_s w + F_f \), or the following nominal (referred to the platform surface area) stress, corresponding to the visible-invisible state transition:
where $\sigma_f$ is the friction stress. The invisible-visible state transition will take place when the platform is moved in the opposite direction, applying a nominal stress:

\[
\sigma_{i\to v} = \sigma_f - \frac{3}{\pi} \frac{\varphi \gamma_s w}{d_s^2}
\]

Considering plausible values of $E_{NT} \approx 1$ TPa, $n_{NT} \approx 0$, $d_s \approx 50$ nm, $\gamma_s = \gamma_{vdW} \approx 0.01$ N/m (van der Waals), from eq. (5) we estimate a contact transversal width of $w \approx 0.8$ nm; and thus taking $\sigma_f \approx 0$ and noting that $\varphi = \sigma_c^{(theo)}/\sigma_{NT}^{(theo)} \approx 10^{-4}$ in an invisible cable, we deduce from eqs. (6) $\sigma_{v\to i} = |\sigma_{i\to v}| \approx 0.3$ Pa. In this case, a negative value of $\sigma_{i\to v}$ suggests that this transition would be spontaneous (friction and gravity neglected) and the existence of a “solid capillary effect”, that could be used for building nanoelevators. Thus the mechanism is very efficient requiring a small control pressure. Evidently the moving platform could be fixed at one of the two terminal ends (the bottom one in Fig. 1), to have a visible cable wound on a ratchet and becoming invisible when unwound by applying a cable stress $\sigma_{v\to i}$. On the other hand, when the cable is invisible could spontaneously return to the visible state, by an instability towards the nanotube configuration reported in Figure 1b. From the equilibrium of this configuration we can estimate the minimum value of $p/L$ required to avoid the spontaneous invisible-visible transition (from $2\sigma \text{Atan} \theta = \sigma_s \Delta w$, see Fig. 1b):

\[
\sigma_{v\to i} = \sigma_f + \frac{3}{\pi} \frac{\varphi \gamma_s w}{d_s^2}
\]
where $\sigma_s$ is the theoretical strength of the surface interaction and $\sigma$ is the applied stress in the aligned nanotubes. For example, considering $d_+ \approx 50$ nm, $\sigma_s = \sigma_{vdW} \approx 1$ MPa, $\sigma = 1$ GPa, $\Delta x \approx w \approx 1$ nm, we deduce $p/L \bigg|_{\min} \approx 2.5 \times 10^{-7}$ ($k \approx 3927$); since for an invisible cable $p \approx 5$ µm, $L_{\max} \approx 20$ m. Note that $L$ here has the physical meaning of distance between two adjacent platforms and is not necessarily the total cable length: more spacer platforms along the cable can thus avoid spontaneous invisible-visible transition, even for smaller applied tension and longer cables.

3. Conclusions

Summarizing, in this paper we have shown that high strength over density ratio invisible cables could be produced in the near future, thanks to carbon nanotube technology. The cable transport will not be problematic in the visible state, whereas the visible-invisible transition can be easily controlled by the reversible and efficient proposed mechanism. Moreover, the spontaneous invisible-visible transition can be avoided by a sufficiently large cable tension and/or number of spacer platforms. Defects (in addition to the complete invisibility demand) could pose limitations to their (nominal) strength, but strongly increasable substituting the invisibility with a less restrictive transparency demand; however their strength to density ratio remains huge.

References

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