Doctoral School of Materials Engineering
Methods of statistical and numerical analysis (integrated course). Part I
Final test - April 2nd 2009

□ **Exercise 1** **Points 3**

A conducting line on an IC chip is $L = (2.8 \pm 0.1)$ mm long and has a rectangular cross section $a \times b$, where $a = (1.00 \pm 0.01)$ μm and $b = (4.00 \pm 0.01)$ μm. A current of intensity $I = (5.120 \pm 0.002)$ mA produces a voltage drop of $V = (101 \pm 1)$ mV across the line. Determine the conductivity

$$\sigma = \frac{LI}{abV}$$

and the corresponding absolute error. Estimate the precision of the result.

□ **Exercise 2** **Points 4**

By assuming that the temperatures below (in K) follow a normal distribution

$$786\,, \quad 790\,, \quad 792\,, \quad 796\,, \quad 784\,, \quad 791\,,$$
$$802\,, \quad 789\,, \quad 783\,, \quad 785\,, \quad 795\,, \quad 782\,,$$

check for the possible presence of an outlier not belonging to the statistical population of the sample.

□ Exercise 3 **Points 5**

We want to check whether the Young modulus E of a polymer follows a normal distribution. To this aim we carry out 200 measurements of Young modulus and compute the relative sample mean $\overline{m}$ and standard deviation s, which can be assumed as estimates of the mean and standard deviation of the whole population, respectively. The results are binned according to the following frequency table:

i	interval of E	empirical frequency
1	$m < \overline{m} - 2.0s$	4
2	$\overline{m} - 2.0s \leq m < \overline{m} - 1.5s$	10
3	$\overline{m} - 1.5s \leq m < \overline{m} - 1.0s$	23
4	$\overline{m} - 1.0s \leq m < \overline{m} - 0.5s$	30
5	$\overline{m} - 0.5s \leq m < \overline{m}$	31
6	$\overline{m} \leq m < \overline{m} + 0.5s$	35
7	$\overline{m} + 0.5s \leq m < \overline{m} + 1.0s$	29
8	$\overline{m} + 1.0s \leq m < \overline{m} + 1.5s$	22
9	$\overline{m} + 1.5s \leq m < \overline{m} + 2.0s$	12
10	$\overline{m} + 2.0s \leq m$	4

Check the hypothesis of the normal distribution with a significance level (a) of 5% and (b) of 1%.

□ Exercise 4 **Points 4**

A random sample of 400 bolts produced by an automatic machine has a mean weight of 7.47 g, with a standard deviation of 0.15 g. We want to determine, for the weight of the bolts:

(a) the confidence interval at a confidence level of 67%;

(b) the confidence interval at a confidence level of 99%.

□ Exercise 5 **Points 4**

Repeated measurements of the electromotive force between the poles of an electric battery have yielded the following results (in V):

i	V_i	i	V_i
1	1.5710	12	1.5703
2	1.5742	13	1.5729
3	1.5751	14	1.5668
4	1.5686	15	1.5725
5	1.5712	16	1.5696
6	1.5734	17	1.5742
7	1.5682	18	1.5711
8	1.5718	19	1.5729
9	1.5737	20	1.5711
10	1.5757	21	1.5689
11	1.5668	22	1.5731

By assuming that the population is normal, calculate the confidence interval of the mean and that of the standard deviation, both at the confidence level of 90%.

□ Exercise 6 **Points 4**

A polymerization process, not yet well standardized, yields polymer samples whose degree of crystallinity c and resistance to traction F vary at random according to a joint probability distribution which can be assumed normal. We wonder whether the two quantities may be correlated. To test the conjecture we carry out 12 measurements on the same number of samples, whose results are summarized in the following table (in arbitrary units):

i	c_i	F_i
1	20.23	8.53
2	8.17	3.82
3	6.01	3.62
4	9.93	6.23
5	13.55	7.57
6	17.89	7.20
7	12.19	6.05
8	1.83	1.10
9	21.72	9.46
10	11.39	5.59
11	3.23	1.66
12	4.34	2.12

Apply Pearson's linear correlation coefficient to check whether the quantities c and F can be regarded as stochastically independent, at a significance level of 5% and 1%. Comment on the physical meaning of the result.

☐ Exercise 7 **Points 6**

The table below collects some experimental measurements of the electrical resistivity ρ (in $10^{-7}\,\Omega\cdot\text{m}$) of a metallic alloy as a function of the absolute temperature T (in K):

k	T_k	ρ_{k1}	ρ_{k2}	ρ_{k3}	ρ_{k4}
1	440	1.55	1.80	1.75	1.65
2	600	2.10	1.95	2.05	2.00
3	800	2.40	2.30	2.50	×
4	1000	2.60	2.90	2.85	2.95
5	1200	3.25	3.30	3.40	3.45
6	1300	3.65	3.70	3.75	×
7	1400	3.75	3.85	3.80	3.95

The random error on the temperatures T_k is negligible, whereas the resistivity data ρ_k are independent normal random variables with the same standard deviation σ (i.e., the system is homoscedastic).

Find:

(*i*) the least squares regression straight line of the form

$$\rho = \mu + \kappa(T - \overline{T}),$$

where $\overline{T}$ is the arithmetic mean of the temperatures;

(*ii*) the 95% confidence intervals of the parameter μ and of the slope κ;

(*iii*) the 95% confidence region for predictions;

(*iv*) the 95% confidence interval for the value of ρ as predicted at $T = 1170$ K;

(*v*) the goodness of fit of the regression model if $\sigma = 0.09$ is the common standard deviation of all the data ρ.

☐ Exercise 8 **Points 4**

11 samples of a conductive polymer are subjected to a thermal treatment at the temperature of 320 K. Further 14 samples of the same material are subjected to a treatment of the same duration, but at a temperature of 370 K. The electrical conductivity of all the samples is then measured, obtaining the results in the table (data in $10^{-1}\mathrm{S}\cdot\mathrm{m}^{-1}$):

$T = 320$ K	$T = 370$ K
5.52	6.33
6.30	5.89
5.00	5.25
5.80	7.70
6.45	7.66
4.38	5.44
5.25	7.10
6.02	7.48
4.77	5.95
5.68	5.05
6.13	6.47
	6.10
	5.24
	7.41

Assuming that the populations are normal, and after having checked whether the relative variances can be regarded as equal or not, determine with a significance level of 5% if the temperature of the thermal treatment has a significant effect on the electrical conductivity of the material.

☐ Exercise 9 **Points 4**

A set of 8 samples is subjected to a physico-chemical treatment which is supposed to affect significantly the thermal conductivity of the material constituting the samples. The thermal conductivity is measured for each sample before and after the treatment, providing the following results (in $W \cdot m^{-1} \cdot K^{-1}$):

before the treatment	after the treatment
1.02	1.16
0.95	1.04
0.73	0.85
0.96	1.10
0.79	1.09
1.01	0.93
0.42	0.95
0.87	1.03

We want to check, with a significance level of 2%, the hypothesis that the means μ_1 and μ_2 of the measured quantity are the same before and after the treatment, by assuming that the populations are normal.

Remark The sufficient grade corresponds to 18 points