**Doctoral School of Materials Engineering**
**Methods of statistical and numerical analysis**
**(integrated course). Part I**
**Final test - June 9th 2009**

**□ Exercise 1 Points 3**

The Young's modulus $E$ of a material is related to the Poisson's ratio $\nu$ and the shear modulus $G$ of the same material by means of the general relationship

$$E = 2G(1 + \nu) .$$

Let the Poisson's ratio and the shear modulus of a tin-copper alloy be given by:

$$\nu = 0.34 \pm 0.01 \qquad\qquad G = (44.7 \pm 0.1)\,\mathrm{GPa} .$$

Determine the Young's modulus of the material and the corresponding absolute error. Estimate the precision of the result.

**□ Exercise 2 Points 4**

The following density data of an expanded polystyrene foam are assumed to obey a normal distribution (data in $\mathrm{kg\,m^{-3}}$)

$$\begin{array}{cccccc} 199\,, & 184\,, & 186\,, & 192\,, & 196\,, & 191\,, \\ 190\,, & 183\,, & 185\,, & 195\,, & 189\,, & 182\,. \end{array}$$

Check for the possible presence of a result not belonging to the statistical population of the sample.

**□ Exercise 3 Points 5**

In order to check whether the shear modulus $G$ of a polymer follows a normal distribution, we carry out 300 measurements of shear modulus and compute the relative sample mean $\overline{m}$ and standard deviation $s$. Due to the large size of the sample, $\overline{m}$ and $s$ can be assumed as good estimates of the mean and standard deviation of the whole population, respectively. The binned results are shown in the following frequency table:

| i | interval of $G$ | empirical frequency |
|---|---|---|
| 1 | $m < \overline{m} - 2.0s$ | 6 |
| 2 | $\overline{m} - 2.0s \le m < \overline{m} - 1.5s$ | 15 |
| 3 | $\overline{m} - 1.5s \le m < \overline{m} - 1.0s$ | 35 |
| 4 | $\overline{m} - 1.0s \le m < \overline{m} - 0.5s$ | 45 |
| 5 | $\overline{m} - 0.5s \le m < \overline{m}$ | 48 |
| 6 | $\overline{m} \le m < \overline{m} + 0.5s$ | 52 |
| 7 | $\overline{m} + 0.5s \le m < \overline{m} + 1.0s$ | 43 |
| 8 | $\overline{m} + 1.0s \le m < \overline{m} + 1.5s$ | 33 |
| 9 | $\overline{m} + 1.5s \le m < \overline{m} + 2.0s$ | 16 |
| 10 | $\overline{m} + 2.0s \le m$ | 7 |

Test the hypothesis of the normal distribution with a significance level: ($a$) of 10%; ($b$) of 5%.

## □ Exercise 4 **Points 4**

A random sample of 500 screws produced by an automatic machine has a mean length of 7.45 mm, with a standard deviation of 0.05 mm. Determine the confidence interval for the length of the screws:

($a$) at a confidence level of 67%;

($b$) at a confidence level of 99%.

## □ Exercise 5 **Points 4**

The electrical conductivity of a metallic alloy has been repeatedly measured, providing the data table below (in $10^6\,\mathrm{S\,m^{-1}}$):

| $i$ | $\sigma_i$ | $i$ | $\sigma_i$ |
|---|---|---|---|
| 1 | 25.711 | 12 | 25.707 |
| 2 | 25.741 | 13 | 25.729 |
| 3 | 25.752 | 14 | 25.656 |
| 4 | 25.687 | 15 | 25.720 |
| 5 | 25.713 | 16 | 25.695 |
| 6 | 25.734 | 17 | 25.742 |
| 7 | 25.681 | 18 | 25.753 |
| 8 | 25.717 | 19 | 25.743 |
| 9 | 25.739 | 20 | 25.713 |
| 10 | 25.753 | 21 | 25.688 |
| 11 | 25.671 | 22 | 25.730 |

By assuming a normal population, compute the confidence interval of the mean and that of the standard deviation, both at the confidence level of 90%.

### ☐ Exercise 6 **Points 4**

A chemical process, not yet well standardized, provides product samples whose degree of crystallinity $c$ and thermal conductivity $k$ vary at random according to a normal joint probability distribution. We guess that the two quantities may be correlated. To test the conjecture, 14 measurements are carried out on the same number of samples and the results are summarized in the table below (in arbitrary units):

| $i$ | $c_i$ | $k_i$ |
|---|---|---|
| 1 | 16.11 | 8.02 |
| 2 | 9.17 | 3.82 |
| 3 | 7.01 | 3.62 |
| 4 | 10.93 | 6.23 |
| 5 | 14.55 | 7.57 |
| 6 | 18.89 | 7.20 |
| 7 | 13.19 | 6.05 |
| 8 | 2.83 | 1.10 |
| 9 | 22.72 | 9.46 |
| 10 | 12.39 | 5.59 |
| 11 | 4.23 | 1.66 |
| 12 | 5.34 | 2.12 |
| 13 | 19.32 | 9.75 |
| 14 | 17.47 | 7.68 |

Applying Pearson's linear correlation coefficient, check whether $c$ and $k$ can be regarded as stochastically independent at a significance level ($a$) of 5% and ($b$) of 1%. Comment on the physical meaning of the result.

## ☐ Exercise 7 **Points 6**

The table below collects some experimental measurements of the dynamic viscosity $\mu$ (in $10^{-4}$ Pa s) of pure water as a function of the temperature $T$ (in $^o$C):

| $k$ | $T_k$ | $\mu_{k1}$ | $\mu_{k2}$ | $\mu_{k3}$ | $\mu_{k4}$ |
|---|---|---|---|---|---|
| 1 | 10 | 13.08 | 13.00 | 12.92 | 13.20 |
| 2 | 20 | 10.03 | 9.992 | 10.06 | 10.18 |
| 3 | 30 | 7.978 | 7.970 | 7.989 | $\times$ |
| 4 | 40 | 6.531 | 6.539 | 6.520 | 6.524 |
| 5 | 50 | 5.471 | 5.454 | 5.465 | 5.482 |
| 6 | 60 | 4.668 | 4.652 | 4.679 | $\times$ |
| 7 | 70 | 4.044 | 4.032 | 4.048 | 4.055 |

The temperatures $T_k$ are affected by no significant error, while the viscosity data $\mu_k$ can be described as independent normal random variables with the same standard deviation $\sigma$.

Determine:

($i$) the least squares regression straight line of the form

$$\mu = \alpha + \beta(T - \overline{T}),$$

where $\overline{T}$ is the mean of the temperatures;

($ii$) the 95% confidence intervals of the regression parameters;

($iii$) the 95% confidence region for predictions;

($iv$) the 95% confidence interval for the value of $\mu$ predicted at $T = 25$ $^o$C;

($v$) the goodness of fit of the regression model if $\sigma = 0.05$ is the common standard deviation of all the data $\mu$.

**☐ Exercise 8 Points 4**

10 samples of a metallic alloy are thermally treated at the temperature of 800 K. Other 14 samples of the same material are subjected to a treatment of the same duration, but at a temperature of 1200 K. The electrical conductivity of all the samples is then measured. The results are summarized in the table below (data in $10^6\,\mathrm{S\,m^{-1}}$):

| $T = 800$ K | $T = 1200$ K |
|---|---|
| 16.30 | 15.89 |
| 15.00 | 15.25 |
| 15.80 | 17.70 |
| 16.45 | 17.66 |
| 14.38 | 15.44 |
| 15.25 | 17.10 |
| 16.02 | 17.48 |
| 14.77 | 15.95 |
| 15.68 | 15.05 |
| 15.52 | 16.32 |
| | 16.47 |
| | 16.10 |
| | 15.24 |
| | 17.41 |

Assuming that the populations are normal, check whether the relative variances can be regarded as equal or not. Determine then, with a significance level of 5%, if the temperature of the thermal treatment significantly affects the electrical conductivity of the material.

**□ Exercise 9** **Points 4**

We guess that a thermal treatment may affect the degree of crystallinity of a polymer. To check our conjecture, the degree of crystallinity of 10 samples is measured prior to and after the treatment. The data are listed below:

| before treatment | after treatment |
|---|---|
| 10.5 | 12.3 |
| 13.0 | 13.2 |
| 15.3 | 16.3 |
| 11.2 | 12.5 |
| 14.6 | 13.9 |
| 12.3 | 14.1 |
| 14.0 | 15.2 |
| 13.2 | 12.8 |
| 14.9 | 15.5 |
| 11.9 | 12.7 |

We want to check, with a significance level of 2%, the hypothesis that the treatment actually affects the degree of cristallinity, by assuming that the populations are normal.

**Remark** The sufficient grade corresponds to 18 points

**Duration of the test: 3 hours**