A simplified model for deep water renewal in Lake Baikal

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ABSTRACT
In deep temperate lakes, thermobaricity (i.e. the decrease in the temperature of maximum density of fresh water with increasing depth) is the crucial physical property governing deep ventilation. If an external forcing is strong enough to mix and move the surface layers downward to a critical depth, thermobaric downwelling could be initiated and a portion of cold and oxygenated surface water freely sinks. The renewal of deep water determines significant effects on the ecobiology of the whole lake. Estimates of the sinking volume extension from the observations are often uncertain, suggesting that developing a modelling tool may be a challenging and worthwhile task.

A simplified one-dimensional numerical model has been developed to analyze the deep downwelling mechanism. Given a profile of the vertical diffusivity, the numerical model solves the reaction-diffusion equation for temperature and any other generic tracer C (e.g. dissolved oxygen, passive tracers, biological tracers). The pressure is assumed as hydrostatic, and a non-linear equation of state is used, which takes into account the effects of both pressure and temperature on water density. The downwelling occurrences (volume, arrival depth) are simulated given the seasonal cycle of surface layer temperature and a stochastic model for external forcing. Hence, variable volumes of surface water are forced to sink down to a given depth, depending on the available external energy input and the instantaneous temperature profile. These intrusions may determine an instability in the density profile that is handled by means of a Lagrangian algorithm, which at every temporal step checks the stability of the water column. Such a model, thanks to its simplicity, requires few input data and low computational efforts that make it suitable to perform long term simulations. Therefore, secular trends could be observed analyzing the numerical results obtained using different climate change scenarios.

The model has been calibrated and validated on the South Basin of Lake Baikal obtaining a good agreement between the numerical results and the temperature.

KEYWORDS
Deep ventilation; Downwelling; Lake Baikal; Thermobaricity; Numerical model.

INTRODUCTION
Deep ventilation in lakes is the renewal of hypolimnetic deep-water by mixing and/or replacement with surface water. This phenomenon shows important effects on the ecobiology of the whole lake since it is capable to enrich the hypolimnetic waters in dissolved oxygen and enhance the nutrients exchange along the entire water column.
Temperate lakes (i.e. lakes in which the surface water temperature passes through the temperature of maximum density $T_{\rho \text{max}}$ twice a year) show two typical thermal stratification profiles: (a) an inverse, stable profile in the uppermost strata of the basin overlying warmer, stratified water during the cold season and (b) a regular stratified profile with temperatures decreasing with depth and always above $T_{\rho \text{max}}$ during the warm season. Shallow lakes are mostly dimictic, that is they are characterized by two periods of global circulation of their water (once in autumn and the other in spring, when the surface water temperature approaches $T_{\rho \text{max}}$). On the contrary, in the deepest temperate lakes typically only the surface layer turns over twice a year, while the deep-water is just occasionally and partially renewed.

In this work, the focus is on deep-water renewal mechanism that occurs in profound, temperate lakes. Lake Baikal (Siberia), the deepest and largest lake in the world in terms of volume, has been assumed as reference basin due to its importance and the dataset available. Observing Lake Baikal, Weiss et al. (1991) explained the deep-water renewal as a consequence of thermobaric instability, that is the property for which $T_{\rho \text{max}}$ of freshwater (nearly 4°C at the atmospheric pressure) decreases with increasing depth. Therefore, if a volume of surface water (colder that the deep water) is pushed sufficiently in depth by an external forcing, beneath a certain depth it results denser than the surrounding water. Thus, it starts to sink freely until it reaches deep water having the same temperature (i.e. density) or until the very bottom of the lake. The depth at which the sinking surface water and the in situ water show the same density is known as compensation depth $h_c$ (Weiss et al., 1991). For typical deep intrusions in Lake Baikal, $h_c$ can be localized at about 250 m depth. For a detailed description of deep ventilation mechanism, see Weiss et al. (1991), Watts and Walker (1995) and Wüest et al. (2005).

Downwellings are likely to happen on occasion of the transition between the two seasonal stratification profiles, when the temperature gradient of the uppermost layer is small, and the energy required to mix and move the surface water volume until $h_c$ is lower. The physical process responsible to provide the energy necessary to initiate deep downwellings has not been clarified completely, and several theories have been proposed: cabling instability of thermal bars (Shimaraev et al, 1993), dense water river inflow (Homann et al., 1997) and wind energy input (Weiss et al., 1991; Killworth et al., 1996; Botte and Kay, 2002; Wüest et al., 2005; Schmid et al., 2008).

In order to investigate the deep-water renewal mechanism and evaluate the extension and frequency of the sinking volumes, many data analyses have been conducted (Hohmann et al., 1997; Ravens et al., 2000; Peeters et al., 2000; Wüest et al., 2005; Schmid et al., 2008) and some numerical simulation adopting 2D or 3D models have been performed (Akitomo et al., 1995; Watts and Walker, 1995; Botte and Kay, 2002). Due to the big extension of Lake Baikal and the complexity of the physical phenomena taking place in its waters, the available data are usually not sufficient to apply complex statistical tools or use sophisticated thermo-hydrodynamic models. As a matter of fact, the main numerical works adopting 2D or 3D models have been performed referring to very schematic and/or partial domains and adopting simplified boundary conditions (in particular with respect to the wind forcing). Under these conditions, it is often difficult and uncertain to obtain a rigorous representation of the processes occurring in the real basin, suggesting the development of simplified numerical models as an interesting alternative that, besides, is perhaps more coherent with the few available data.

Therefore, a simplified one-dimensional model has been developed, which simulates deep ventilation triggered by thermobaricity occurring in profound lakes. The model requires a few data in input that, together with its simplified structure, permits both to reduce the need of
artificial parameterizations and to ensure a fast computational speed. The accuracy in the results is consistent with that of the input data.

In order to verify the consistency of the model, it has been applied to the case of the South Basin of Lake Baikal. The model has been calibrated by comparing numerical results with the available observation data of temperature (by courtesy of Prof. Alfred Wüest and his research team, EAWAG) and other tracers. Successively, the model has been validated through long-term (i.e. centuries) simulations, and the main variables (e.g. mean annual downwelling volume, typical downwelling temperature) have been estimated and compared to literature.

METHODS

A time-dependent, one-dimensional numerical model has been developed in order to study deep-water renewal occurring in profound, temperate lakes. The model solves a reaction-diffusion equation for the temperature by means of the usual backward Euler implicit finite difference scheme. Convection contribute is introduced by using two algorithms handling (a) the convection due to buoyancy within the unstable regions ($N^2<0$, where $N^2$ is the Brunt-Väisälä frequency) of the water column, and (b) the deep convection due to downwelling occurrences.

For the generic tracer $C$, the reaction-diffusion equation solved by the model is

$$\frac{\partial C}{\partial t} = -\frac{1}{\delta} \frac{\partial (S \phi)}{\partial z} + R,$$

where $t$ is the temporal variable, $z$ is the vertical coordinate defined positive downward (i.e. $z$ is the depth), $S$ is the horizontal surface at a fixed depth, $\phi$ is the vertical tracer flux, $D_z$ is the diffusion coefficient, and $R$ is the reaction term describing sources and sinks. In general all the parameters are dependent on the depth.

The non-linear Chen and Millero (1986) equation of state for freshwater has been used, in order to evaluate to high precision (better than $10^{-6}$ g/cm$^3$) the main thermophysical properties of lake water required by the model (e.g. density $\rho$, specific heat capacity $c_p$, thermal expansibility $\alpha$ etc.):

$$\rho = \rho(T,P) = \rho_0 \left(1 - \frac{P}{K} \right)^{-1},$$

where $T$ is the water temperature, $P$ is the pressure, $\rho_0 = \rho_0(T)$ is the reference density at the atmospheric pressure, and $K$ is a function of $T$ and $P$. Following Killworth et al. (1996) and Wüest et al. (2005), water salinity has been assumed to contribute little to the stratification of Lake Baikal, thus its contribution has been neglected in evaluating the thermophysical properties.

The pressure $P$ has been assumed as hydrostatic:

$$\frac{dP}{dz} = \rho g \frac{dT}{dz},$$

where $g$ is the gravitational acceleration. Hydrostatic approximation is convenient and justified to simulate long term dynamics (i.e. longer than the time scale of the single process) in which the instantaneous behavior is not of main interest.

Hereafter, the vertical displacement of water volumes (due to downwelling events and/or to water column stabilization) is assumed to follow an adiabatic path (i.e. the moving volumes do not exchange heat with the surrounding water) according to

$$\frac{dT}{dz} \bigg|_{\text{adiabatic}} = \frac{g \alpha (T + 273.15)}{c_p} = \Gamma.$$
where $\Gamma$ is the adiabatic temperature gradient, which is known to be small, especially near $T_{\rho_{\text{max}}}$, where it vanishes. However, if stratification is weak $\Gamma$ could assume values close to the real temperature gradient, gaining in importance for the thermal stability of the water column (for further details see Landau and Lifshitz, 1987; Osborn and LeBlond, 1974).

The numerical discretization of the physical domain follows a finite volume scheme which divides the water body into $n$ sub-volumes having the same volume (thereby, in the most general case, having different vertical extension according to the hypsometric curve of the basin). The extent (thus, the number) of the sub-volumes is decided a priori trying to find an acceptable compromise between a good resolution of the results and a sufficiently high computational speed. In this work, the South Basin of Lake Baikal (having a depth of 1,461 m and a volume of about 6,300 km$^3$) has been discretized into 159 sub-volumes that have a volume of 40 km$^3$ and are characterized by a minimum, maximum and mean vertical extension of 5 m, 66 m and 9 m, respectively. A staggered grid has been used for the numerical solution of equation (1), in which the variable of the model (the generic tracer $C$) and the main thermophysical variables ($\alpha$, $\rho$ and $c_p$) are defined at the center of each sub-volume, while the diffusion parameter $D_z$ is defined at the sub-volume interfaces. This approach allows one to calculate the fluxes at the sub-volumes’ boundaries, thus implementing a mass-conservative scheme.

The model consists of three main components: (a) an algorithm handling the vertical stabilization in the case of buoyancy instability, (b) a Lagrangian-based algorithm handling the deep downwelling mechanism, and (c) the reaction-diffusion equation solver. Aimed at guaranteeing both little computational cost and good temporal resolution of the results, a time step $\Delta t$ of half a day has been chosen ($\Delta t = 12$ h). Both the reaction-diffusion solver and the stability algorithm handling buoyant mixing are solved with this temporal step. On the contrary, the algorithm handling the deep ventilation mechanism uses a different time step, $\Delta t_{\text{down}}$, a multiple of $\Delta t$, which represents the temporal scale of the downwellings. At each time step $\Delta t$, the model follows the sequence of operations shown in Figure 1.

**Figure 1.** Block diagram of the model representing the sequence of the operations solved at each time step.

**Figure 2.** Sketch for the stability evaluation procedure: stable profile for $\alpha > 0$, $\Delta \rho(T, P)|_{z_{\text{mid}}}=\rho(T_{i+1\text{up}}, P|_{z_{\text{mid}}})-\rho(T_{i\text{down}}, P|_{z_{\text{mid}}}) > 0$. 

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Stability algorithm
At each time step $\Delta t$, the entire water column is tested for static stability by evaluating the density difference $\Delta \rho$ of each couple of neighbor sub-volumes $i$ and $i+1$ (respectively centered in $z_i$ and $z_{i+1}$) at their midpoint ($z_{i\text{ mid}}$). The sub-volumes are ideally shifted towards $z_{i\text{ mid}}$ following the adiabatic transformation $\Gamma$, and two new values of temperature are computed (see Figure 2): the temperature $T_{i+1\text{ up}}$ of the lower sub-volume $(i+1)$ displaced upward, and the temperature $T_{i\text{ down}}$ of the upper sub-volume $(i)$ displaced downward. Therefore, the density of each displaced sub-volume is computed with respect to the new temperature and pressure at $z_{i\text{ mid}}$, and the local stability is evaluated according to the condition

$\begin{align*}
\left. \Delta \rho(T,P) \right|_{z_{i\text{ mid}}} &> 0 \quad \text{stable} \\
\left. \Delta \rho(T,P) \right|_{z_{i\text{ mid}}} &= 0 \quad \text{neutral} \\
\left. \Delta \rho(T,P) \right|_{z_{i\text{ mid}}} &< 0 \quad \text{unstable}
\end{align*}$

(6)

where

$\left. \Delta \rho(T,P) \right|_{z_{i\text{ mid}}} = \rho(T_{i+1\text{ up}}, P(z_{i\text{ mid}})) - \rho(T_{i\text{ down}}, P(z_{i\text{ mid}}))$.

(7)

Starting from the couple showing the higher instability, the whole water column is progressively stabilized by simply inverting the position of the unstable sub-volumes. The rearrangement of the unstable regions is instantaneously performed at each time step. In this way, the vertical convection due to buoyancy is not represented exactly with its actual physical temporal scale (i.e. hours to days). Nevertheless, the overall vertical convection mechanism is suitably reproduced for long term analyses (i.e. weeks to years, the temporal scale investigated in this work).

Downwelling algorithm
To handle the deep downwelling process, a Lagrangian-based algorithm is performed at each downwelling time step $\Delta t_{\text{down}}$. Each sub-volume is temporarily discretized in $m$ homogeneous, smaller parts (hereafter sub-volume parts) having the same value of $C$ as that of the initial sub-volume (in this work each sub-volume, 40 km$^3$, has been divided in 8 parts having a volume of 5 km$^3$). This allows one to model deep ventilation using a finer computational grid, with negligible increase of the computational cost. At each downwelling time step, probabilistic values of the external specific energy input, $e_I$, and the descending volume extension, $V_{\text{down}}$, are assigned by means of a stochastic approach. $e_I$ and $V_{\text{down}}$ are calculated from the wind speed and duration, which are randomly extracted from seasonal (i.e. winter and summer) probabilistic curves that can be constructed on the basis of large datasets. Given the temperature profile at that time, the downwelling temperature $T_d$ and the compensation depth $h_c$ are calculated respectively as: (i) the mean temperature within the active layer (corresponding to $V_{\text{down}}$), and (ii) the depth at which the sinking volume (that follows the adiabatic transformation $\Gamma$ during its descent) shows the same density as the surrounding water. Note that, because of the typical thermal profile and the thermobaric effect, they usually have different temperatures (being the local and the descending water above and below the $T_{\rho \text{ max}}$, respectively). Once $T_d$ and $h_c$ have been computed, the model calculates the energy per unit volume $e_R$ that the descending water volume should have to overcome the buoyancy force and reach $h_c$. A number $p$ of sub-volumes parts having a cumulative volume that matches $V_{\text{down}}$ are displaced downward until a certain depth depending on the comparison between $e_R$ and $e_I$. In case $e_R<e_I$ a shallow downwelling occurs and the $p$ sub-volumes parts are displaced downward to a depth $h< h_c$. In this case the sinking water is lighter than the surrounding water and generates an unstable profile that is stabilized by the stability
algorithm at the beginning of the following time step. On the contrary, if \( e_I > e_R \) a deep event occur and the \( p \) sub-volumes parts are displaced beneath \( h_c \) until local water having the same temperature of the sinking volume is reached or, when the intrusion is heavier than the deep water, until the very bottom of the lake. Once the downwelling is performed, the previous discretization is reestablished, re-combining together the sub-volumes parts in groups of \( m \) elements and computing the mean value of the generic tracer \( C \) for each group.

**Heat equation solver**
The last step is the solution of the reaction-diffusion equation (1) for the temperature and the other tracers (e.g. dissolved oxygen and CFC concentration). The backward Euler implicit scheme has been used, obtaining an unconditionally stable numerical solution. As a consequence, no restriction are needed for \( \Delta t \). In order to guarantee both little computational efforts and good temporal resolution of the results, a \( \Delta t \) of half a day has been chosen.

The boundary conditions at the surface and at the bottom are respectively fixed (a) through the assignment of the superficial value of the tracer \( C \) according to measures (for the temperature) or analytical relationships (for the dissolved oxygen or the CFC’s concentration), and (b) by the imposition of a Neumann condition at the bottom of the lake (geothermal heat flux for \( T \), areal consumption rate for the dissolved oxygen, and no flux condition for CFC). Concerning the reaction term \( R \) in each vertical layer, this is has been calculated differently for each type of tracer: assuming the geothermal heat flux contribution at the sediment-water interface for \( T \), estimating the volumetric and areal consumption rate for the oxygen along the water column, and assuming \( R=0 \) for the CFC.

**RESULTS AND DISCUSSION**
To apply the model to any particular case, it is necessary to first calibrate the main parameters: the seasonal probabilistic curves of wind speed and duration and the diapycnal diffusivity profile. Lake Baikal, the world’s deepest and largest freshwater basin in terms of volume, has been assumed as case study. The model has been applied to the South Basin of the lake, where deep ventilation has been observed and studied during the last decades. The calibration has been performed comparing numerical simulations with measured profiles of temperature and a passive tracer (CFC_{12}) (data from Killworth et al. 1996; Peeters et al., 2000). Successively, the diapycnal diffusivity profile and the stochastic model for the wind forcing have been validated through a long-term (8 centuries) simulation. An additional validation of the model has been obtained by comparing the values of the calibrated parameters (i.e. diapycnal diffusivity profile) and the characteristic variables (i.e. mean annual downwelling volume, typical downwelling temperature) with estimates from literature. The calibration and validation procedures are discussed in detail in Piccolroaz and Toffolon (2011), while in this article the main results of a medium-term simulation are presented. The calibrated model has been applied to simulate a 50 years period, assuming the external conditions to remain the same (i.e. no climate change scenarios). The seasonal probabilistic curves of wind speed and duration have been built from the long-term observation dataset available in Zhelplinsky and Sorokina (1977), while the surface temperature cycle has been obtained on the basis of a 9-year measurements dataset (2000-2009, courtesy of Prof. A. Wüest and his research team). In Figure 3, simulated temperature profiles are compared with measurements (from the available dataset, 2000-2009) in two typical winter and summer periods. Due to the stochastic approach used to model the external forcing, numerical results have been averaged over the last 10 years of simulation. The model allows for a quantitative estimate of the main characteristics of the deep ventilation mechanisms. The range of
temperature and volumes of deep downwelling events in Lake Baikal are reported in Table 1, where results from model are presented together with literature estimations. As a whole, the results are coherent with the existing measurements, which are however affected by non-negligible uncertainties. Finally, it is worth noting that the present model can be used to derive complete statistics of the downwelling events, which are not reported here for lack of space but will be examined in detail in Piccolroaz and Toffolon (2011).

![Figure 3. Comparison between simulated and measured temperature profiles in winter (15 February 2000-2009, left) and summer (15 September 2000-2009, right). Numerical results have been averaged over the last 10 years of simulation.](image)

### Table 1.
Comparison between numerical results and literature estimations of the main variables characterizing deep ventilation in Lake Baikal (all the variables refer to downwellings beneath 1400 m depth): mean annual downwelling volume $V_{down}$ and typical downwelling temperature $T_{down}$.

<table>
<thead>
<tr>
<th></th>
<th>$T_{down}$ (°C)</th>
<th>$V_{down}$ (km³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present model</td>
<td>3.17±0.11</td>
<td>80±30</td>
</tr>
<tr>
<td>Peeters et al., 2000</td>
<td>-</td>
<td>110</td>
</tr>
<tr>
<td>Wüest et al., 2005</td>
<td>3.15±3.27</td>
<td>10±30</td>
</tr>
<tr>
<td>Schmid et al., 2008</td>
<td>3.03±3.28</td>
<td>50±100 (winter season)</td>
</tr>
</tbody>
</table>

### CONCLUSIONS
A simplified, one-dimensional numerical model has been presented that is suitable to analyze deep ventilation occurring in profound lakes. The model numerically computes the evolution of the vertical temperature profiles (and any other tracer) taking into account the stability of the water column and the occurrences of deep water intrusions. The model handles a few data in input (according to the reduced information often available for large basins) and shows a simplified structure. Together these features ensure a significant computational time saving, while the accuracy of the results is consistent with that of the input data.
In order to face the lack of wind data, a stochastic approach has been developed for the wind forcing, that is based on seasonal probabilistic curves for wind speed and duration. Long-term observational datasets are required to construct such probabilistic distribution. The model has been calibrated and validated performing medium- and long-term simulations (from decades to centuries) over the South Basin of Lake Baikal. The numerical results show good agreement with measurements and literature estimates, and the model seems to simulate the deep ventilation phenomenon accurately.

Due to the considerable computational speed, the model is suitable to analyze the future behavior of the lake and its response to climate change scenarios, by performing long time simulations (i.e. hundreds of years). Moreover, further analysis could be focused on characterizing the implications of deep ventilation for the vertical distribution of dissolved oxygen, nutrients, biological components and other tracers.

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