LARGE AMPLITUDE EFFECTS ON TIDAL AMPLIFICATION
IN CONVERGENT ESTUARIES

M. Toffolon 1, G. Vignoli 1, M. Tubino 1

ABSTRACT: In this paper we study the propagation of a tidal wave in convergent estuaries. In the last few years several contributions have been proposed to investigate this subject, which mainly consist of analytical models based on the assumption that the ratio $\varepsilon$ between the tidal amplitude and the mean water depth is small. We have developed a simple one-dimensional numerical model and found that, provided its value is not vanishingly small, the above ratio may affect significantly the marginal conditions for tide amplification. In the present work, the balance between the damping effect of friction and the amplification due to channel convergence is investigated in terms of external parameters. Furthermore, a new formulation for the scale of velocity is introduced. The dependence of the marginal conditions on the parameter $\varepsilon$ is determined: it appears that, in terms of the relevant dimensionless parameters which account for the effect of friction and convergence, critical curves can be given the form of power laws whose coefficients depend on $\varepsilon$.

1 INTRODUCTION

The hydrodynamics of estuaries is strongly affected by the geometrical characteristics of the channel. Although many other elements may be relevant, in the present work we restrict the analysis to a widespread class of tidal inlets, namely the well-mixed estuarine channels. This kind of morphological large-scale elements includes those estuaries and lagoon channels where the tidal forcing is so strong that stratification does not occur. The absence of the salt wedge allows one to consider a constant density of water and to describe the flow field using the usual equations of single-phase fluid.

Understanding the hydrodynamics of tidal channels is relevant for many environmental issues. For the evaluation of the consequences of both natural and anthropogenic modifications it is essential to describe the role of several basic factors (length of the estuary, friction, channel convergence, bed altimetry, river discharge) on the properties of the tidal wave (Toffolon, 2002). Moreover, hydrodynamics is the basic ingredient which drives the morphological development of tidal channels, as also discussed in the companion paper (Todeschini et al. 2003).

The problem of the propagation of the tidal wave in convergent channels has been tackled by several authors in recent years (e.g. Friedrichs and Aubrey 1994; Friedrichs et al. 1998; Lanzoni and Seminara 1998), following the contribution of Jay (1991) who has first revisited the problem of tide amplification due to channel convergence, which was originally investigated by Green (1837). Though these theories provide valuable results for the comprehension of the basic mechanisms, they mostly rely on the assumption that some parameters can be considered small.

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so that the mathematical description of the system can be simplified through a linear or weakly non-linear analysis. In the present work we try to remove the above restriction and investigate large amplitude effects on tidal wave propagation tackling the fully non-linear problem through a numerical model.

In tidal channels the driving force is the water oscillation imposed at mouth of the channel, which is connected with the outer sea. Almost all the non-linear terms in the governing equations for the flow field are proportional to the ratio

\[ \varepsilon = \frac{a_0}{D_0} \]  

between the tidal amplitude \(a_0\) and the average water depth \(D_0\), where the subscript \(0\) refers to the values at the mouth of the channel. The analytical solutions mentioned above assume the ratio \(\varepsilon\) to be small enough such that its effect is negligible at the leading order of approximation. Unfortunately, in many real estuaries \(\varepsilon\) can reach relatively large values as shown in Table 1.

<table>
<thead>
<tr>
<th>Estuary</th>
<th>(a_0) ([m])</th>
<th>(D_0) ([m])</th>
<th>(\varepsilon)</th>
<th>Estuary</th>
<th>(a_0) ([m])</th>
<th>(D_0) ([m])</th>
<th>(\varepsilon)</th>
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<td>0.04</td>
<td>Severn</td>
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<td>15.0</td>
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<td>7.5</td>
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<td>Thames</td>
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<td>10.0</td>
<td>0.10</td>
<td>Hoogly</td>
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<td>5.9</td>
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<td>0.20</td>
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Table 1: Values of the amplitude ratio \(\varepsilon\) for various estuaries, evaluated using the data reported by Lanzoni and Seminara (1998).

The present work is specifically devoted to identify the marginal conditions for tide amplification in convergent estuaries. In this case the amplification of the tidal wave due to the decrease of channel cross-sectional area is counteracted by frictional dissipation. The dynamical balance between these two effects is investigated introducing suitable dimensionless parameters. The main result of present analysis is that critical curves which define the marginal conditions for tidal wave amplification along the estuary can be given the form of power laws in terms of the above dimensionless parameters.

Notice that tidal wave generates overtides along its propagation (for a review, see Parker 1991). For the sake of simplicity in the present work we disregard external overtides and force the system with a purely sinusoidal semi-diurnal \(M_2\) tide at the mouth of the estuary. Overtides at the mouth of the estuary, like the quarter-diurnal \(M_4\), are present in nature when the offshore shelf is wide and flat; their determination is beyond the aims of the present analysis. Notice that the presence of a wider wave spectrum at the boundary condition may affect the global behaviour, but it introduces a larger number of degrees of freedom in the analysis.

### 2 FORMULATION OF THE PROBLEM

We consider a tidal channel with varying width and depth and we investigate the propagation of the tidal wave through a one-dimensional cross-sectionally averaged model, with longitudinal...
coordinate \( x \) directed landward, starting from the mouth of the estuary. The seaward boundary condition is given, in terms of the free surface, by the sea level, which is assumed to be determined by the tidal oscillation without any influence of the internal response of the estuary. We also assume that a rectangular cross-section is suitable to describe, as a first approximation, the behaviour of a real section of the channel.

The above assumptions may be rather strong, since they imply that the model is unable to account for topographically driven effects on the flow field as well as to include the role of shallow areas adjacent to the main channel. A more refined approach would imply the matching of the one-dimensional (global) model with a local model, say a model with a characteristic length scale of the order of channel width, where the above effects can be considered.

Figure 1: Sketch of the estuary and basic notation.

The sketch of the geometry of the idealized tidal channel is represented in Figure 1. The width of the channel is assumed to be an exponentially decreasing function of the longitudinal coordinate \( x \), according to the relationship:

\[
B = B_\infty + (B_0 - B_\infty) \exp \left( -\frac{x}{L_b} \right) \tag{2}
\]

where \( L_b \) is the convergence length and the asymptotic width \( B_\infty \) is included to set a minimum width landward also when convergence is strong and the estuary is long. The standard one-dimensional shallow water equations are used, which read

\[
\begin{align*}
\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q^2}{\Omega} \right) + g\frac{\partial H}{\partial x} + \frac{Q|Q|}{\Omega C_h^2 R_h} &= 0, \\
\frac{\partial \Omega}{\partial t} + \frac{\partial Q}{\partial x} &= 0,
\end{align*}
\]

where \( t \) is time, \( Q \) is the water discharge, \( \Omega \) is the area of the cross section, \( H \) is the free surface elevation, \( C_h \) is the dimensionless Chézy coefficient, \( R_h \) the hydraulic radius, \( g \) is gravity and the water depth \( D = H - \eta \) is defined in terms of bottom elevation \( \eta \).

As for the longitudinal bottom profile, it is common to observe that in tidal channels the flow depth decreases landward. Prandle (1991) found that the behaviour of the width and depth of real estuaries can be described in terms of power laws. Bottom profiles may be chosen.
analytically (e.g. linear, exponential) or evaluated through morphological models (Lanzoni and Seminara, 2002). In the present work, as a first step of the analysis, we assume the bed to be horizontal.

We note that channel convergence can be given the form

$$\frac{1}{B} \frac{dB}{dx} = - \frac{1}{L_b} \frac{B - B_\infty}{B} \simeq - \frac{1}{L_b},$$

which is valid when the asymptotic width $B_\infty$ is much smaller than the actual width $B(x)$. In the simulations presented herein we have assumed a wide channel and set $B_\infty = 0$ and $R_h \simeq D$.

In this case the dependence of the solution on the actual width of the channel can be ruled out, as it can be readily seen from equations (3-4) rewritten in terms of the cross-sectionally averaged velocity $U = Q/\Omega$:

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + g \frac{\partial H}{\partial x} + \frac{U |U|}{C_h^2 D} = 0,$$

$$\frac{\partial D}{\partial t} + D \frac{\partial U}{\partial x} + U \frac{\partial D}{\partial x} - \frac{UD}{L_b} = 0,$$

The system (3-4) is written and solved numerically in semi-conservative form in terms of the variables $H$ and $Q$, in order to enhance the conservation of mass and momentum. The system is discretized through finite differences and solved numerically using the explicit MacCormack method. The numerical method is second order accurate both in space and in time. The stability condition requires that the Courant-Friedrichs-Levi number does not exceed the unity. Since the tidal wave tends to break during its propagation, mainly due to the effect of the frictional term and convergence, an algorithm to avoid spurious oscillations must be adopted. In present analysis we use the TVD filter (Total Variation Diminishing), which has been successfully applied in the simulation of the hydrodynamics of free-surface flows (see, for instance, Garcia-Navarro et al. 1992).

The boundary condition seaward is straightforward: we impose that the free surface level is given by the semi-diurnal $M_2$ tide, whose period and peak amplitude at the mouth are denoted by $T_0$ and $a_0$, respectively. The second boundary condition is the landward relationship between free surface level and water discharge. Such a condition is difficult to be posed because many estuaries are the final reaches of rivers. Frequently the presence of a peculiar geometrical configuration induces a reflection of the tidal wave. Two limit cases can be recognized: i) the reflecting barrier and ii) the transparent condition. The former situation corresponds to impose a vanishing discharge at the landward end of the computational domain, which determines a complete reflection of the wave. The latter condition refers to a situation where no obstacles are present and the tidal wave exits from the computational domain without being deformed or reflected. In this case, the model reproduces a long estuary, for which the effect of the boundary condition is not relevant. Notice that, as pointed out by Friedrichs and Aubrey (1994), the landward boundary condition is less important in the case of strongly convergent channels.

3 EXTERNAL PARAMETERS

The relevant dimensionless parameters are defined in terms of external quantities, namely the amplitude of tidal excursion at the mouth, the reference values of channel width and depth, say the values at a given representative section of the estuary, the characteristic lengths of width and depth variations along the estuary, and the friction factor. Moreover, the length of the estuary itself, which embodies the influence of the landward boundary condition, often plays a crucial role in the definition of the flow field.
We consider the reference velocity as an internal quantity, since it is only determined once the other parameters are given. Consequently, all the parameters defined in the following do not involve directly the velocity, but they only imply an evaluation of a reference value of velocity in terms of external quantities. A first relevant external parameter is the dimensionless tidal amplitude $\varepsilon$, defined in (1), which plays an important role both from geometrical and kinematic point of view.

In the frictionless case, in a infinitely long channel with constant width, the solution is a wave function whose celerity is $c_0 = \sqrt{gD_0}$; hence, a frictionless wavelength can be defined as

$$L_g = \sqrt{gD_0 T_0}, \quad (8)$$

which can be taken as a reference length scale. The frictionless velocity $U_g$ can be expressed as a function of external parameters in the following way:

$$U_g = \varepsilon \sqrt{gD_0} = \frac{a_0 L_g}{D_0 T_0}. \quad (9)$$

Lanzoni and Seminara (1998) introduce the ratio between frictional terms $R$ and inertial terms $S$ in the momentum equation

$$\frac{R}{S} = \frac{U_0 T_0}{C_h^2 D_0} \quad (10)$$

to take into account the effect of dissipation (see also Parker 1991). In order to define the above parameter in terms of external variables, since we do not know a priori the scale of velocity $U_0$, we use equations (8) and (9) to write

$$\chi = \frac{a_0 L_g}{C_h^2 D_0} = \varepsilon \frac{L_g}{C_h^2 D_0}. \quad (11)$$

Please note that $\chi$ is linearly proportional to $\varepsilon$; thus high values of $\varepsilon$ can be related in almost all cases to frictionally dominated estuaries.

Finally, to account for the effect of width variation along the estuary a dimensionless convergence ratio can be defined in the form:

$$\gamma = \frac{L_g}{L_b}, \quad (12)$$

where $L_b$ is the convergence length defined in (2).

Recently Toffolon (2002) has proposed a simplified formulation to evaluate the reference velocity at the mouth of the estuary in terms of external parameters. An analysis of the relative importance of the various terms in the momentum and continuity equations, through dimensional considerations, leads to the following relationships for the scale of velocity $U_0$, defined as the algebraic mean of peak values of flood and ebb velocity at the mouth of the estuary:

$$U_0 = \frac{1}{\chi} \left( \Delta - \frac{1}{\Delta} \frac{\hat{\gamma}}{\hat{\chi}} \right) U_g, \quad (13)$$

where

$$\Delta = \left[ 1 + \sqrt{1 + \left( \frac{\hat{\gamma}}{\hat{\chi}} \right)^3} \right]^{\frac{1}{3}}, \quad \hat{\chi} = \left( \frac{\chi}{\pi} \right)^{1/3}, \quad \hat{\gamma} = \frac{\gamma - 4}{3\pi}. \quad (14)$$

Equation (13) includes and improves relationships which have been already proposed in the asymptotic limit of strongly convergent and weakly dissipative estuaries and for its dual
Figure 2: Contour plot of the ratio $U_0/U_g$ between the reference velocity and the frictionless velocity in terms of the dimensionless parameters $\chi$ and $\gamma$ (transparent boundary condition landward, $D_0 = 10m$). Solid lines: theoretical prediction obtained through equation (13); broken lines: numerical results corresponding to different values of $\varepsilon$ (dotted lines: $\varepsilon = 0.1$, dash-dot lines: $\varepsilon = 0.2$, dashed lines: $\varepsilon = 0.3$).

case (see, for instance, Lanzoni and Seminara 1998). Also note that (13) is valid provided the frictionless limit $U_0 < U_g$ is satisfied and for constant depth channels. In Figure 2 the proposed analytical scale for velocity (solid lines) is compared with the results of numerical simulations (broken lines). Numerical results correspond to different values of $\varepsilon$, which ranges between 0.1 and 0.3, such that a relatively wide range of values of $\chi$ can be covered while the friction coefficient keeps within a realistic range of values. In fact, for a given $\varepsilon$ numerical results cover only a limited range of $\chi$, which changes for different values of $D_0$. It appears that the comparison is fairly satisfactory within a wide range of values of the parameters. It also appears that dependence of numerical results on tidal amplitude, i.e. on $\varepsilon$, becomes significant as the degree of convergence of the estuary increases.

4 MARGINAL CONDITIONS FOR TIDE AMPLIFICATION

The marginal conditions for the amplification of the wave amplitude are defined by those values of the relevant parameters for which the tidal wave does not amplify nor decay during its propagation, within a reach of the estuary relatively close to its mouth. As pointed out before the dynamic balance which governs the amplification of a tidal wave essentially involves convergence and friction; hence, theoretical considerations suggest that marginal conditions are likely to be expressed in terms of the degree of convergence $\gamma$ and the friction to inertia ratio $\chi$.

A first attempt to characterize the behaviour of a tidal wave in convergent geometries is due to Green (1837), who determined the well known relationship

$$\frac{a(x)}{a_0} = \left( \frac{B_0}{B(x)} \right)^{1/2} \left( \frac{D_0}{D(x)} \right)^{1/4}. \tag{15}$$

The Green’s law is based on energy conservation considerations and relies on two unrealistic
assumptions, namely that energy dissipation is negligible and convergence is much weaker than the tidal wavelength. In particular, the frictionless character of the relationship does not allow one to describe the wave damping. According to (15) any degree of channel convergence or decrease of flow depth should result into an amplification of the tidal wave, which is obviously not true in real estuaries.

Figure 3: Marginal conditions for the amplification of tidal amplitude in the $\chi$-$\gamma$ plane, for different values of $\varepsilon$, as obtained through the numerical model. The interpolating power laws are reported in the plot with the corresponding correlation coefficient $R^2$.

Jay (1991), Friedrichs and Aubrey (1994) (see also Friedrichs et al. 1998) and Lanzoni and Seminara (1998) have proposed suitable extensions of Green’s law that take into account also the role of friction. Their analytical approaches are based on the assumption that the parameter $\varepsilon$ is relatively small. As discussed in the preceding section the order of magnitude of $\chi$ is strictly related to $\varepsilon$; hence such condition corresponds to consider weakly dissipative estuaries. On the other hand, in strongly dissipative cases, tide propagation has to be treated as a strongly non-linear process. The above theories, which more or less implicitly consider marginal conditions for tide amplification, lead to relationships for the marginal state that can be cast in the following form:

$$\gamma = k\chi^m. \quad (16)$$

For weakly convergent and weakly dissipative estuaries Lanzoni and Seminara (1998) and Friedrichs et al. (1998) have found a linear dependence, hence $m=1$. On the other hand, for the case of strongly convergent channels, Friedrichs and Aubrey (1994) have found that marginal conditions are attained when the actual celerity $c$ of the tidal wave is equal to the frictionless celerity $c_0$. Adapting their relationship to our notation, we find

$$\gamma = \frac{4}{\sqrt{3}}\chi^{1/2}. \quad (17)$$

Hence for strongly convergent estuaries the exponent of power law (16) reduces to $m = 0.5$.

In the present work the marginal conditions for tide amplification have been determined through numerical experiments by running each numerical simulation until a tidally averaged
equilibrium state was reached by the system and comparing the resulting tidal amplitude at a
given section with that imposed at the mouth. An iterative algorithm has been developed in
order to obtain, within a given tolerance, the value of the length \( L \) for which the difference was
minimized. Simulations have been performed placing a reflective barrier landward; however,
the length of the estuary was always large enough to avoid any influence on the solution. More
than 200 simulations have been conducted in a wide range of values of the parameters, namely
\( D_0 \in [2.5m \div 50m] \), \( \varepsilon \in [0.005 \div 0.6] \), \( C_\chi \in [10 \div 30] \), paying particular attention to the choice
of typical conditions of real estuaries.

![Figure 4: The coefficients \( m \) (left) and \( k \) (right) of equation 16 as functions of the amplitude ratio \( \varepsilon \).](image)

The marginal conditions obtained numerically are plotted in Figure 3 in terms of the dimen-
sionless ratio between friction and inertia \( \chi \) and the degree of convergence \( \gamma \), for different values
of the amplitude ratio \( \varepsilon \). Below the numerical points the wave is damped during its propagation
landward, while above them it is amplified. It is worth noticing that the larger are the dissipative
effects along the estuary (large \( \chi \)), the stronger is the required degree of convergence to achieve
the marginal conditions. On the contrary, a relatively weak variation of channel geometry can
produce wave amplification in weakly dissipative estuaries.

From Figure 3 it appears that numerical points corresponding to a given value of \( \varepsilon \) can be
fitted fairly well through power law curves of the form (16), whose coefficients \( m \) and \( k \) only
depend on the parameter \( \varepsilon \). The above dependence embodies the effect of the finite amplitude of
the tidal wave on its amplification. The numerical findings are summarized in Figure 4 where \( m \)
and \( k \) are plotted as a function of \( \varepsilon \): it is shown that the exponent \( m \) depends strongly on finite
amplitude effects such that its value decreases sharply as \( \varepsilon \) increases and reaches an almost
constant value, nearly equal to 0.6, for relatively large values of \( \varepsilon \).

Numerical results for the weakly convergent and weakly dissipative case are represented in
more detail in Figure 5. As pointed out before, analytical results suggest that in this case a linear
relationship should hold, with \( m=1 \). Numerical results conform to this behaviour only for very
small values of \( \varepsilon \). For commonly observed values of the tidal range within this class of estuaries
the power law is non-linear as shown in the figure. This is even clearer when considering that,
as shown in Figure 4, the tendency of \( m \) towards unity is almost vertical. Notice that the linear
solution would be satisfactory in the case of an horizontal asymptotic trend of the curve towards
\( m=1 \). Also notice that the landward boundary condition becomes more and more important in
the numerical model as we approach the frictionless limit, hence when we consider very small
values of \( \chi \) and \( \varepsilon \).

In the case of strong convergence a significant influence of the tidal amplitude on wave
characteristics can be expected along with a substantial deviation from the linear behaviour. In
this case the approximate analytical solution (see equation 17) leads to a value of the exponent
Figure 5: Marginal conditions for tidal wave amplification in the $\chi$-$\gamma$ plane, for small values of the amplitude ratio $\varepsilon$. The interpolating power laws are reported in the plot with the corresponding correlation coefficient $R^2$. $m = 0.5$ which is fairly close to the numerical result ($m \simeq 0.6$) within a wide range of values of $\varepsilon$, namely those which are typically encountered in real estuaries according to the data reported in Table 1. Notice, however, that the coefficient $k \simeq 2.31$ of the relationship (17) does not fall within the range (0.6-1.2) obtained through numerical simulations and reported in Figure 4.

It is worth noticing that if we introduce a modified parameter

$$\tilde{\chi}(\chi, \varepsilon) = k(\varepsilon)\chi^m(\varepsilon), \quad (18)$$

using the values of the coefficients $k$ and $m$ given in Figure 4, the numerical points which define the marginal conditions for tide amplification collapse on a single logarithmic plot as shown in Figure 6.

Figure 6: Marginal conditions in terms of the modified dimensionless parameter $\tilde{\chi}$ defined in equation (18). The interpolating power law is reported with the corresponding correlation coefficient $R^2$.

5 CONCLUSIONS

Simplified analytical models have been proposed in the recent past to investigate the propagation of the tidal wave in convergent estuaries, which are based on the assumption that the tidal
amplitude can be considered small with respect to the averaged depth; this is not the case in many tidal environments. The results of the numerical model proposed herein suggest that nonlinearities associated with finite amplitude effects strongly affect tidal propagation. In particular, we have shown that marginal conditions for tidal wave amplification can be given the form of power laws in terms of the relevant dimensionless parameters which account for the effects of friction, inertia and convergence. Such laws exhibit a strong dependence on the dimensionless tidal amplitude \( \varepsilon \) such that the linear behaviour predicted by the analytical solution of Lanzoni and Seminara (1998) in case of weakly dissipative and weakly convergent estuaries is recovered only for very small values of \( \varepsilon \), while the strongly convergent solution of Friedrichs and Aubrey (1994) is never attained precisely. It is worth noticing that the geometry of the tidal channel has been strongly simplified in the present analysis which essentially retains only the effect of variable width. Hence the role of several aspects typical of estuarine morphology has been neglected. Some of them may play a crucial role in the definition of marginal conditions, like depth variations, included in the original formulation of Green (1837), the presence of intertidal areas and the effect of river discharge.

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