Long-term morphological evolution of funnel-shape tide-dominated estuaries
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Abstract. We investigate the long-term morphological evolution of a tidal channel through a one-dimensional numerical model. We restrict our attention to the case of tide-dominated estuaries, which are usually characterized by a funnel shape, and neglect the effect of intertidal areas and river discharge, imposing a closed boundary at the landward end. If the estuary is relatively short and weakly convergent the equilibrium bottom profile extends over the entire length of the estuary, whereas a beach is formed inside the domain when the initial length of the channel exceeds a threshold value. Hence, it is possible to define an intrinsic equilibrium length as the distance between the beach and the mouth. In our analysis we examine how such estuarine length, which is independent of the physical dimension imposed to the system, is affected by three main parameters, namely channel convergence, tidal amplitude at the mouth and friction. We show that the degree of convergence plays a crucial role, as the analysis of real estuaries seems to confirm: a strong degree of convergence implies shorter equilibrium lengths. We also show that increasing the tidal amplitude at the mouth or the channel friction produces shorter equilibrium profiles. Numerical results suggest that tidal asymmetries vanish as the system approaches the final equilibrium state.

1. Introduction
Estuaries are particular environments, characterized by an intense human presence but, at the same time, by a very rich ecosystem. Understanding their morphology poses a very interesting, though quite complex, issue that can be tackled at different spatial and temporal scales. The long-term evolution of these systems, say that occurring over a time scale of the order of centuries, is determined by the internal morphological changes (i.e. those associated with the mutual interactions between the flow field and the bottom surface) and by the variations of the external conditions (sea level rise, subsidence, tectonic uplift). As a result, the relative contribution of the above factors can hardly be distinguished in the evolutionary process observed in natural contexts.

The aim of this work is to investigate the long-term equilibrium bottom profiles of tidal channels. However, in our analysis we discard the variation of external forcings and investigate those processes that rule the internal response. More specifically we analyze the dependence of the equilibrium configuration on the relevant physical parameters that characterize the tide and the channel geometry, namely channel convergence, tidal amplitude at the mouth and friction. In this contribution the attention
is turned to the case of well-mixed, tide-dominated estuaries which are typically characterized by a funnel shape in their planimetric view. Channel convergence strongly affects the hydrodynamics of such estuaries (e.g. Jay, 1991; Friedrichs and Aubrey, 1994; Lanzoni and Seminara, 1998; Savenije and Veling, 2005; Toffolon et al., 2006). If the width decreases exponentially and the initial bottom profile is horizontal, the channel is typically flood dominated in a large part of its length and the net flux of sediment is mainly directed landward [Wells, 1995; Lanzoni and Seminara, 2002]. This asymmetry during a tidal cycle causes a net landward sediment flux, as it has been observed in many tide-dominated estuaries like the Ord River estuary in Australia (Wright et al., 1973), the Salmon River estuary in Canada (Dalrymple et al., 1990) and the Severn estuary in the UK (Murray and Hawkins, 1977). The net sediment transport, which is mainly related to the degree of asymmetry between the flood and the ebb peak values of flow velocity, determines the long term erosional and depositional processes in the tidal channel. The above scenario is confirmed also by the experimental observations of Tambroni et al. [2005].

Several models have been proposed so far to study the morphological evolution of tidal channels and their equilibrium configuration at a global scale. In almost all cases, the formulation is one-dimensional, despite this kind of models is not able to deal with estuarine circulations and stratification dynamics; nevertheless, the necessity to run long-term simulations or to deal with simplified models does not allow one to consider two- or three-dimensional models. For the fairly idealized case of tidal channels with constant width, Schutteelaars and de Swart [2000] have shown that the equilibrium profiles for a short estuary are characterized by an almost constant bed slope, while for longer estuaries the bottom profiles show an upward concavity near the entrance of the channel, such as the depth increases landward in the region immediately upstream the mouth. Such behavior, which is not commonly encountered in real estuaries, is mainly related to the effect of the seaward boundary condition adopted in their model, where the bed level is given a prescribed value. More realistic profiles have been obtained through the evolutionary model of Hibma et al. [2003] who have relaxed the above constraint. Afterward, Lanzoni and Seminara [2002] have highlighted the role of channel convergence on bed profile of tidal channels, showing a comparison between the simulated morphological evolution of a convergent channel and a constant width channel. However their analysis was not aimed at providing a comprehensive picture of the role of the different factors affecting the equilibrium profile.

A further attempt to tackle the problem by means of an analytical simplified model has been pursued by Prandle [2003], who has found an analytical relationship for the equilibrium bottom profile in the limit of strong convergence. At a smaller length scale, Pritchard et al. [2002] have used a one-dimensional numerical model to determine the long-term configuration of intertidal mudflats.

Like in previous works, in our analysis we assume that channel width does not change in time (a recent attempt to investigate the combined effect of width changes is due to Todeschini et al. [2005]). We also neglect the role of intertidal areas and the fresh water upstream contribution. We tackle the problem by means of a one-dimensional numerical model in order to more easily retain key non-linearities that are crucial for a suitable estimation of the residual terms governing the morphological evolution [Toffolon et al., 2006]. As a primary analysis, we consider the smallest number of parameters characterizing
the problem, though retaining the main physical mechanisms driving the morphological evolution and influencing the equilibrium profiles. Thus, we characterize tidal forcing by the only semidiurnal constituent and neglect the presence of overtones at the mouth. At the landward boundary we assume a reflecting barrier condition with no river flow, which strictly applies only to tide-dominated estuaries where the tidal prism is much larger than the river discharge. In this case tidal dominance establishes in most part of the estuary and governs the morphological behavior, though riverine currents and the associated sediment input may not be negligible in the upstream part as the cross-sectional area of the channel becomes smaller [Dalrymple et al., 1992].

In Section 2 the model is introduced along with the main assumptions and the numerical scheme is briefly described. In Section 3 the influence of the hydrodynamics on the residual sediment transport is discussed, some examples of morphological evolution are shown and the overall results are interpreted as a function of the main parameters. Then, in Section 4 the model results for the estuarine length are compared with data from real estuaries and with the analytical solution proposed by Prandle [2003]. Finally, the salient characteristics of the model are reviewed in Section 5.

2. Formulation of the problem

We investigate the long-term morphological evolution of a tide-dominated channel with fixed banks; the adopted schematization is shown in Figure 1 along with the relevant notations. The one-dimensional model is based on the assumption that the channel cross-section can be considered rectangular and the role of intertidal areas can be neglected as a first approximation.

The typical funnel shape of the channel is described by an exponentially decreasing function of the longitudinal coordinate $x$ starting from the estuary mouth, as it is commonly assumed by many authors (e.g. Ippen, 1966; Parker, 1991; Friedrichs and Aubrey, 1994; Lanzoni and Seminara, 1998); the width $B$ is then written as:

$$B = B_0 \exp \left( -\frac{x}{L_b} \right) , $$

where $L_b$ is the convergence length and $B_0$ the width at the channel mouth. Note that hereafter the subscript 0 indicates quantities at the mouth of the estuary in the initial configuration.

Following the standard one-dimensional shallow water approach, the continuity and momentum equations for water flow read:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 , $$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q^2}{A} \right) + gA \frac{\partial H}{\partial x} + gj = 0 , $$

where $t$ is time, $Q$ the water discharge, $A$ the area of the cross section, $H$ the free surface oscillation and $g$ is gravity; furthermore, the frictional term $j$ is defined as

$$j = \frac{U|U|}{k_s R_h^{\frac{4}{3}}} , $$

where $k_s$, the Gauckler-Strickler coefficient, is the inverse
of the Manning roughness coefficient, $R_h$ is the hydraulic radius (defined as the ratio between the channel cross section $A$ and the wetted perimeter $B + 2D$, where $D$ is the water depth) and

$$U = \frac{Q}{A}$$

indicates the flow velocity.

The system is forced by a purely sinusoidal semi-diurnal $M_2$ tide at the mouth of the channel,

$$H(t)|_{x=0} = a_0 \cos(2\pi \frac{t}{T_0}),$$

where $a_0$ is the tidal amplitude at the mouth and $T_0$ the tidal period. We note that the role of overtides in forcing sea surface elevation at the mouth is not considered in the analysis, while internally generated overtides are accounted for by our numerical solution which retains the fully nonlinear character of the governing equations (2)-(3).

The morphodynamic evolution of the channel is driven by the continuity equation for the sediments which reads:

$$(1 - p)B \frac{\partial \eta}{\partial t} + \frac{\partial (Bq_s)}{\partial x} = 0.$$  \hspace{1cm} (7)

In (7) $p$ is sediment porosity, $q_s$ is the sediment flux per unit width and $\eta = H - D$ is the bottom elevation. In our model the sediment flux $q_s$ is evaluated using the relationship proposed by Engelund and Hansen [1967]:

$$q_s = \sqrt{g \Delta d_s^3} \cdot 0.05 C_h^{2} \frac{\theta^{5/2}}{2},$$  \hspace{1cm} (8)

where $d_s$ is the characteristic particle diameter, $\Delta$ is the relative density of sandy sediments with respect to water, $\theta = U^2 / (C_h^2 g \Delta d_s)$ is the Shields parameter and $C_h = k_s R_h^{1/6} g^{-1/2}$ is the dimensionless Chezy coefficient. The relationship (8) computes the total sediment load, including the suspended load. Alternatively, the sediment flux could be evaluated by considering the separate contributions of the bed load and the suspended flux and solving for the latter a suitable transport equation to compute suspended sediment concentration. This kind of approach would be required to reproduce correctly the settling lag associated with suspended load, which may be important in natural estuaries, in particular for short channels. In the numerical results presented in the following paragraphs we assume a sediment diameter $d_s = 10^{-4}$ m (fine sand), a porosity $p = 0.3$ and $\Delta = 1.65$.

From equation (7) it is possible to derive a morphological timescale for bed evolution $T_b$. Using the initial depth $D_0$, as a scale for the variations of bed elevation $\eta$, and a reference sediment flux per unit width $q_{s0}$ at the mouth, such timescale reads

$$T_b = \frac{(1 - p) D_0 L_0}{q_{s0}}.$$  \hspace{1cm} (9)

In order to evaluate the tidally averaged magnitude of the reference sediment flux $q_{s0}$, we temporarily assume a sinusoidal velocity $U = U_0 \sin(2\pi t / T_0)$ at the mouth and neglect variations of the depth during the tidal cycle; using such expression and averaging equation (8) we
obtain a suitable estimate of the scale of sediment flux in the following form:

\[ q_{so} = 0.05 \frac{16}{15 \pi} \frac{U_0^5}{k_s \Delta^2 d_s \sqrt{gD_0}}. \] (10)

We note that the timescale \( T_b \) is relative to the initial conditions because it depends on the initial values of velocity and depth. It is also interesting to remark that \( T_b \) is typically some orders of magnitude greater than the tidal period \( T_0 \), which implies that the morphological evolution occurs on a fairly long period of time if compared with the tidal period [see also Schuttelaars and de Swart 1996]. Hence, in order to discuss the behavior of the bottom evolution, one can simplify the problem by expressing the sediment continuity equation (7) in terms of the tidal averaged values:

\[ \frac{\partial \langle \eta \rangle}{\partial t} = -\frac{1}{(1-p)B} \frac{\partial (B \langle q_s \rangle)}{\partial x}, \] (11)

where angle brackets indicate average over a tidal cycle.

The condition \( T_b \gg T_0 \) also allows one to decouple and solve separately the hydrodynamics from the morphodynamic problem.

The differential system (2)-(3), written in semi-conservative form in terms of the variables \( Q \) and \( H \), is discretized through finite differences and solved numerically using the explicit McCormack method [Garcia-Navarro et al., 1992; Lanzoni and Seminara, 2002]. This is a two-step predictor-corrector method characterized by a second order accuracy both in space and in time provided that the usual Courant-Friedrichs-Levi stability condition is respected. A TVD (Total Variation Diminishing) scheme is applied in order to remove the spurious oscillations around discontinuities, which may arise since the tidal wave tends to break during its propagation due to friction and convergence. This filter introduces numerical diffusion only in the points around the discontinuities, where the accuracy of the method reduces to first order. The sediment continuity equation (7) is discretized through finite differences and solved numerically using a first-order upwind method, with the same time step imposed by the CFL condition for the hydrodynamic problem.

Two physical boundary conditions are required for the solution of the hydrodynamic problem. At the sea boundary we suppose the free surface to be only determined by the outer sea without being influenced by the wave propagation into the channel. At the landward boundary the river discharge is supposed to be negligible and a reflecting barrier condition is imposed. Note that the McCormack method requires two further boundary conditions, which are implemented numerically using a first-order in space, first-order in time extrapolation procedure [Hirsch, 1990].

In order to compute the bottom evolution, a further condition must be imposed. During the flood phase we assume that the sediment flux entering at the mouth is given by the transport estimated through the relationship \( (8) \) in terms of the local instantaneous value of flow velocity. In the ebb phase we assume vanishing sediment input at the landward end.

Finally, a suitable wetting-drying routine is required, because the morphological evolution of a tide-dominated estuary in idealized numerical simulations is often characterized by the occurrence of a sediment front that migrates landward and tends to emerge [Lanzoni and Sem-
inara, 2002]. Thus, some cells within the estuary may
undergo a drainage process during the ebb phase, while
they are submerged during the flood phase. In order
to prevent the possibility that negative depths may oc-
cur during the calculation, a simple procedure is adopted
such that at every time step we exclude from the compu-
tational domain those cells where the free surface level is
lower than a threshold value above the bottom elevation;
the last wet cell becomes the last active cell at each time
step. The beach can be located at the end or inside the
domain: in the latter case we exclude from the computa-
tion all the succeeding cells in the landward direction. At
a subsequent time step these cells can be flooded again,
since the free surface is subjected to periodic oscillation;
in this case, we reintroduce into the computational do-
main those cells where the water depth is larger than the
threshold value.

Such simplified procedure, as well as the various sim-
plifying assumptions described above (in particular the
use of an algebraic sediment transport monomial closure
and the neglect of the river input and of the role of in-
tertidal areas) greatly limit the range of applicability of
the model and make it unable to reproduce estuarine
processes that may be relevant, particularly on short-
intermediate time and spatial scales. In this respect the
present model is less refined than other existing models
[Schuttelaars and de Swart, 2000; Lanzoni and Seminara,
2002, e.g]. However the adoption of a much simpler for-
mulation of the problem is highly desirable for long-term
dynamical simulations. Furthermore, a comparison per-
formed between present numerical results and those ob-
tained through the more refined model of Lanzoni and
Seminara [2002] (whose details are omitted here for the
sake of brevity) indicates that the adopted simplifications
are not crucial for the simulation of the long-term behav-
or and the definition of the equilibrium bottom profiles.

3. Results

According to the results of the numerical simulations,
the morphological evolution of an idealized tidal channel
can be described as follows. (i) The bottom elevation
tends to rise where the longitudinal gradient of the resid-
ual sediment flux, $B(q_s)$, is negative, as described by (11);
in particular, starting from an initial horizontal bed pro-
file, a sediment front is formed where the absolute value of
the gradient is larger. (ii) The front migrates landward
and amplifies until it reaches the last section, where it
may be reflected. (iii) After a period of time of the order
of hundreds of years, the system tends to an equilibrium
configuration, which is characterized by a bottom profile
displaying an upward concavity (Figure 2a).

The above behavior, which is typical of short channels,
has been also documented in the two examples reported
[Lanzoni and Seminara [2002]. In this case the physical
length imposed to the system represents a constraint in
the morphological evolution of an estuarine channel. In
fact, when the channel is relatively short (Figure 2a), the
asymptotic configuration is characterized by the forma-
tion of a beach at the landward end, such that the final
length coincides with the imposed length of the estuary
(this condition is used in the following to define a “short
estuary”). The resulting equilibrium bottom profiles dis-
play the same shape as of those obtained by Schuttelaars
and de Swart [2000] for short channels.

On the other hand, in a longer channel (Figure 2b)
the sediment front moves landward and emerges at a cer-
tain distance from the mouth. This condition generally
hinders the further migration of the front. In this case
the tide-dominated part of the estuary occupies only a
fraction of the total length of the channel. Therefore
we define as “long estuary” a tidal channel whose initial
length is large enough not to influence the morphologi-
cal evolution of the system; consequently the final length
is inherently related to the macroscopic characteristics of
the channel (convergence, friction) and of the forcing tide
and is determined by the position of the beach inside the
estuary (see also Todeschini et al., 2003).

In the latter case we define the final equilibrium length
of the estuary, \( L_e \), as the distance between the mouth
and the position of the beach. Such definition is not
trivial since the equilibrium configuration is reached only
asymptotically and in terms of the residual sediment
transport. A static equilibrium condition could be ob-
tained only by imposing a threshold value for the Shields
parameter in the sediment flux estimation and a thresh-
old limit for particle suspension. This may be reasonable
for more weakly tidal estuaries, but the corresponding
velocity is significantly smaller than the typical values
observed in macrotidal estuaries, for example [Friedrichs,
1995]. In the case of Engelund and Hansen formula (8),
the equilibrium can only be dynamical. In particular
it occurs when the residual value of the total sediment
transport \( B(q_s) \) equals everywhere the river sediment in-
flow, if it is present, as suggested by equation (11), while
it vanishes in the case of a closed channel. This is an
asymptotic process as it can be seen in Figure 3, where
the temporal evolution of the residual sediment flux \( \langle q_s \rangle \)
is plotted at the mouth and in the middle of the estuary.
The time required to reach the equilibrium configuration
is usually of the order of a thousand years; in the ex-
ample reported in Figure 3 the morphological time scale
\( T_b \) exceeds 500 years and the equilibrium configuration
is reached in about \( 6 T_b \). Actually, the final state is a
quasi-equilibrium configuration for which it is necessary
to define a threshold value for the bottom variation in a
given time period below which equilibrium is assumed to
be reached and the numerical simulation is stopped. In
all the results presented in this work such threshold value
is fixed to \( 10^{-2} \text{mm/year} \).

Before analyzing in detail the morphological evolution,
it is useful to discuss the influence of the hydrodynam-
ics on the sediment transport. A flood-dominance typ-
ically characterizes tide-dominated estuaries, where the
channel convergence can be very strong and determines
a distortion of the tidal wave. This can be seen from the
distributions along the estuary of the velocity \( U \) and the
sediment flux \( q_s \) at the beginning of the simulation, which
are shown in Figure 4. It is interesting to note that the
position of the initial deposit in Figure 2a corresponds to
the section where the quantity \( -B\partial\langle q_s \rangle/\partial x \) attains its
maximum value (see Figure 4).

On the other hand, the asymmetry between flood and
ebb phase almost disappears at every point along the
estuary at equilibrium. In fact, given the strongly non-
linear dependence of the sediment transport upon the ve-
locity \( q_s \sim U^5 \) in the case of the monomial relationship
(8), the residual sediment transport \( B(q_s) \) mostly de-
pends on the peaks of velocity. Thus, the system evolves
towards the equilibrium configuration by reducing the
difference (asymmetry) between ebb and flood velocities,
which implies a reduction of the gradient of \( B(q_s) \), as
shown in Figure 4.

This tendency towards a symmetric configuration is
clearly displayed in the behavior of the velocity and the
sediment flux during one tidal cycle. At the beginning
of the simulation (Figure 5) the strong asymmetry of the
velocity causes a large difference in the sediment trans-
port during flood and ebb phases, which is more pronounced inside the channel (e.g. \( x = 0.5 L_0 \)) than at the mouth (\( x = 0 \)). On the contrary, at equilibrium velocity and sediment transport tend to be symmetrical both at the mouth and in the middle of the estuary (Figure 6); furthermore, numerical results suggest that the differences among the various sections along the channel are reduced not only in terms of the peak values but also in terms of the time behavior of velocity and sediment flux. Moreover the peak values of velocity keep almost constant along the estuary, as it is observed in many real estuaries [Friedrichs, 1995].

The channel convergence can determine amplification or damping of the tidal wave [Jay, 1991; Savenije and Veling, 2005; Toffolon et al., 2006, e.g.] and enhances the distortion of the tidal wave and the flood-dominance, as it has been shown by Friedrichs and Aubrey [1994] and Lanzoni and Seminara [1998] among others. The influence of the degree of convergence clearly emerges when the peaks and the residual values of the velocity \( U \) and of the sediment flux \( B(q_s) \) are plotted at the initial stage of evolution, when the bed is still horizontal (Figure 7).

The more convergent is the channel, the more the tidal amplitude grows with the wave propagation; moreover the flood-dominated character is enhanced over the entire length of the estuary. When the channel is weakly convergent, the tidal wave tends to be damped during its propagation and both the peak values of the velocity and the sediment flux monotonically decrease landward. On the contrary, when the convergence becomes strong enough, they show a peak inside the estuary and a decreasing trend landward. At equilibrium (Figure 8) the velocity peaks are almost constant in that part of the channel that is not directly influenced by the sloping beach, and the residual sediment transport becomes negligible. Furthermore, the tidal surface amplitude tends to become more uniformly distributed along the length of near equilibrium tidal channels, because the sloping bed reduces the role of wave reflection landward.

The morphological evolution of a tidal channel is a direct consequence of its hydrodynamics, being driven by the longitudinal gradient of the residual sediment discharge, which essentially depends on the asymmetry of velocity distribution. In particular, the equilibrium conditions are achieved when tidal asymmetry vanishes. In this respect the problem of assessing the equilibrium bottom profile is mainly a hydrodynamic problem and therefore it is not crucially affected by the choice of the sediment transport closure adopted in the model.

The parameters that influence the equilibrium configuration are manyfold. In Figure 9 the effect of the degree of convergence of the channel (\( L_b^{-1} \)) is shown by comparing the equilibrium bottom profiles for different values of \( L_b \). In short channels, an increase of the convergence length implies a larger depth at the mouth and consequently a larger bottom slope, while it has no effect on the equilibrium length that is forced by the physical dimension imposed to the system (Figure 9a). On the other hand, increasing the convergence degree in longer channels determines shorter equilibrium lengths. When the final length, \( L_e \), is smaller than the initial one, \( L_0 \), the latter becomes unessential; in fact, given the same value of \( L_0 \), the equilibrium profiles for different values of \( L_0 \) are nearly identical, as shown in Figure 10. In this way we can define an intrinsic equilibrium length \( L_e \) as the length that the estuary would tend to assume in the absence of a boundary constraint. Moreover, when the initial length \( L_0 \) is shorter than the above equilibrium length \( L_e \), the final configuration is that typical of the
case of short estuary.

It is worth mentioning that a first definition of ‘maximum embayment length’ was introduced by Schutelaars and de Swart [2000], who found that in channels longer than a certain value the equilibrium profiles were characterized by a sediment deposit at the landward boundary; the resulting maximum length depends on the tidal forcing and the frictionless wavelength. However, since the latter depends on the value of bottom elevation at the mouth, which is imposed in their model, the equilibrium configuration is not free, but results from such controversial boundary condition.

Figure 11 summarizes the outcomes of the numerical simulations corresponding to different values of the convergence length, $L_b$, and of the physical length imposed to the system in the initial configuration. Each point corresponds to the result of a simulation characterized by a given degree of convergence; the points on each line are characterized by the same value of $L_b$ and by an increasing value of the physical dimension of the domain $L_0$ as we move rightward. Note that in weakly convergent channels the sediment front does not always emerge. However, a suitable equilibrium length can be defined also in this case in terms of the distance of the leading edge of bottom profile from the mouth. When the initial channel is not sufficiently long, that is for small values of $L_0$, the equilibrium length $L_e$ coincides with the physical dimension imposed to the system: the corresponding points fall on the bisector line of the graph. On the contrary, when the channel is long enough, the system is allowed to reach the intrinsic length $\hat{L}_e$, which is independent of the physical dimension of the domain and mainly depends on the degree of convergence of the channel: the stronger is channel convergence (i.e. small values of $L_b$), the shorter is the resulting equilibrium length $\hat{L}_e$ (Figure 11).

We note that the results presented above are not influenced by the initial conditions, like the initial depth $D_0$ and the initial bottom slope. Indeed, these parameters play a negligible role on the final equilibrium configuration, though they are important in the transient phase of the evolution and affect the time required to reach the equilibrium state.

Apart from the estuarine funnelling, other parameters affect the equilibrium morphology of the channel, like friction and tidal forcing; their influence is more pronounced as the degree of convergence decreases. For instance, in weakly convergent channels, different values of the friction coefficient $k_s$ lead to different equilibrium lengths $\hat{L}_e$ (Figure 12a), while this influence almost disappears in strongly convergent channels (Figure 12b). The influence of the external tidal forcing is quite strong since it controls the scale of velocity in the estuary: a larger tidal amplitude implies a larger scour at the mouth of the channel and a more pronounced bottom slope (Figure 13a), which determines a decrease in the equilibrium length $L_e$ (Figure 13b).

Finally, we analyze the average slope of the equilibrium bottom profiles. We use a rough definition of the slope $S = (\eta_{\text{max}} - \eta_{\text{min}})/L_e$, where $L_e$ is the final length, $\eta_{\text{max}}$ is the bed elevation at the head of the sediment front and $\eta_{\text{min}}$ the bed elevation at the estuary mouth, which normally corresponds to the deepest point of the profile. The results are plotted in Figure 14 as a function of the convergence length $L_b$ (left plot) and of the tidal amplitude $a_0$ (Figure 14a), for different values of $k_s$.

We can observe that steeper slopes are associated with stronger tidal forcing, whereas the effect of the convergence degree does not seem to be relevant (Figure 14b). Also friction has a mild influence and rough channel seem...
to have slightly larger slopes.

4. Discussion

The main output of the present work is the recognition that for given channel geometry and tidal forcing the channel reaches an inherent equilibrium length which is mainly governed by the degree of convergence. Understanding the precise mechanisms that lead to the disappearance of tidal asymmetries with the morphological evolution of the channel and determine the system response at equilibrium is not obvious and will deserve further analysis. However, we may note that, given the same tidal forcing, increasing channel convergence leads to a reduction of the velocity amplitude at the mouth (see Figures 7 and 8), while it induces less pronounced scouring effects. On the other hand, the average bed slope at equilibrium does not seem to be appreciably affected by channel convergence (Figure 14a), while it is much more sensitive to the variation of the tidal forcing (Figure 14b). As a result a smaller depth establishes at the mouth (Figure 9) and consequently the channel is shorter when convergence is strong. It is also worth noting that the reduction of both the peak velocity (which is nearly constant along the estuary) and the average flow depth at equilibrium in more convergent estuaries is consistent with the reduction of the tidal prism associated with increasing channel convergence.

The dependence of the equilibrium length upon friction coefficient is not so pronounced. Present results suggest that two counteracting effects determine the role of friction. Smaller values of $k_s$ (i.e., stronger friction) imply larger values of bed shear stress, which induce a larger depth at the mouth (Figure 12a). However, the average bed slope also increases with friction (the effect is similar to that of tidal forcing shown in Figure 14b). The latter effect prevails and leads to a reduction of the equilibrium length for decreasing $k_s$.

It may be of interest to discuss our results in the light of the recent solution of Prandle [2003], who proposes a simplified analytical relationship for the equilibrium bottom profile which reads

$$\langle D \rangle = \left[ \frac{5}{4} \sqrt{\frac{a_0}{2 \pi T_0}} \frac{2 \pi f}{2g^{1/4}} (L_P - x) \right]^{4/5},$$

where $\langle D \rangle \approx -\eta$ is the tidally averaged depth, $f = \frac{g}{(k_s^* \langle D \rangle)^{1/3}}$ is the friction parameter, and $r = 1.46$ comes from the linearization of the frictional term (or $r = 1.33$ according to Prandle [2004]). The estuarine length $L_P$ can be easily derived from (12); in our notation it reads

$$L_P = \left( \frac{2}{5} \frac{2^{3/4} \sqrt{T_0}}{g^{1/4} \sqrt{\pi r}} \right) \frac{k_s \langle D_m \rangle^{17/12}}{\sqrt{a_0}},$$

where $\langle D_m \rangle$ is the tidally averaged depth at the mouth.

One of the main assumptions used by Prandle [2003] to find (12)-(13) is to consider a triangular estuarine cross-sections with constant slope of the banks (i.e. $B = \alpha \langle D \rangle$ with constant $\alpha$). In this way the degree of convergence is directly related to the bottom profile, differently from our model where the former is assigned and the latter is calculated. Considering that the relationship for the length $L_p$ does not consider the explicit role of the degree of convergence, and that the depth at the mouth is not an...
independent variable because it is the result of bottom
evolution, according to (13) the estuarine length mainly
deeps on the two parameters, friction and tidal ampli-
tude, considered in our analysis, in addition to channel
convergence. It is interesting to note that the analytical
relationship (13) reproduces the same dependence of the
estuarine length on such parameters found through our
numerical solution. In spite of the evident differences be-
tween the two models, the qualitative agreement between
them seems to confirm the hydrodynamic nature of the
problem of the equilibrium profiles.

It is also of interest to check if the channel lengths pre-
dicted by the numerical simulations are comparable with
those resulting from empirical observations of the geo-
metrical dimensions of real estuaries. In Table 1 some
characteristic values of \( L_e \) are reported, along with the
other main geometrical parameters [Lanzoni and Sem-
inara, 1998]. Despite the uncertainties related with a
reliable definition of the length in those cases where the
estuary is not closed landward but it receives a non neg-
ligible input from an upstream river, such data can be
used to analyze, at least qualitatively, the effect of the
parameters under investigation. In Figure 15 the estu-
arine lengths \( L_e \) are plotted against the corresponding
convergence lengths \( L_b \). The data are subdivided among
three classes of estuary, depending on the tidal ampli-
tude at the mouth. We note that, as an overall behav-
ior, the length of real estuaries tends to grow with the
convergence length (i.e. it reduces in more convergent
channels), as it is also suggested by the power law inter-
polation of the data reproduced in the figure (in the same
plot present results with a value of the tidal amplitude
\( a_0 = 2 \) m and friction coefficient \( k_s = 40 \) m\(^{1/3}\)s\(^{-1}\) are re-
ported for a comparison, see also Figure 12). Then, it
can be argued that the very nature of exponential con-
vergence means that highly convergent estuaries in na-
ture must be shorter than weakly convergent estuaries.

Moreover, the estuarine length increases with decreasing
values of the tidal amplitude, as it is also suggested by
our numerical solution.

It is worth noting that the qualitative agreement be-
tween numerical predictions and field data is satisfactory
despite the fact that many estuaries in Table 1 do not sat-
isfy the assumptions introduced in our idealized model.
In particular, some of the estuaries lying farther from the
interpolating curve in Figure 15 are known to be partially
mixed (e.g. Columbia, Delaware, Hudson, Potomac) or
are strongly affected by river inflow. The fact that a qual-
itative general tendency is reproduced by our simplified
model suggests that the equilibrium dynamics of natural
estuaries does not strongly depend on the details of the
model formulation, but it is mainly related to the hydro-
dynamic requirement that velocities need to be almost
symmetrical during the tidal cycle in order to reduce the
residual sediment transport.

5. Conclusions

In this work we have investigated the long-term mor-
phological evolution of convergent estuaries with the use
of a relatively simple one-dimensional numerical model,
which neglects the presence of intertidal areas and in
which only the bed is considered erodible. Since the mor-
phological equilibrium is dynamical, the corresponding
bottom profile is defined as the configuration in which
the tidally averaged bottom elevation attains a constant
value. Given the Engelund and Hansen transport formula
(8), this condition is achieved only asymptotically. For
the initial conditions considered here, the typical time
The scale required to achieve quasi-equilibrium conditions is fairly long, say of the order of centuries or millennia. The long-term erosion/deposition process is determined by the residual sediment transport, which is mainly related to the degree of asymmetry between the flood and the ebb peak values of flow velocity, due to the non-linear dependence of sediment transport on flow velocity [Dronkers, 1986]. The channel convergence causes a distortion of the tidal wave and forces the sediments to move towards the inner part of the estuary. The morphological evolution stops when the residual sediment transport vanishes: the process goes on until the asymmetry tends to disappear, thus an almost symmetrical velocity is a necessary condition to have equilibrium. The most important parameter influencing such process is found to be the degree of convergence.

We have shown that the physical length of the channel may represent a constraint for the morphological evolution. In fact, if the initial length is shorter than a threshold value, which decreases for increasing degrees of convergence, the final configuration is only determined by the available length. Otherwise, the emersion of the sediments accumulated within the estuary determines a sort of barrier for the tidal wave propagation. In this case the resulting length is inherently related to channel convergence, as one may expects given its influence on the asymmetry of velocity.

Therefore, it is possible to define an intrinsic equilibrium length of an estuary, which is supposed to be the final configuration of a free morphological evolution when the outer forcing is constant, sea level rise is negligible and anthropogenic influence is not considered. The main finding of our analysis is that such length is forced by the planimetric configuration, say the estuary funnelling; strongly convergent estuaries are expected to be shorter than weakly convergent channels. The equilibrium length is influenced also by the other parameters that characterize the system, as the friction coefficient and the tidal forcing.

We have shown that increasing the tidal amplitude at the mouth or the channel friction (decreasing $k_s$) tends to produce shorter equilibrium profiles. Such behavior conforms to that predicted by the simplified analytical solution proposed by Prandle [2003]. Furthermore, the agreement between predicted equilibrium lengths and the range of the typical lengths of real estuaries is remarkable, though field data are affected by large uncertainties due to external long-term variations that may strongly affect the morphological evolution.

Finally we note that, despite its simplicity, the model allows one to gain some clarification about the role of different aspects (channel convergence, external forcing, friction) on the long-term morphodynamics of tidal channels. Further ingredients should be added, like the freshwater discharge and the presence of intertidal areas, but a systematic analysis of their role is beyond the scope of the present work. As a concluding remark, we note that a thorough calibration of long-term numerical models through field data is almost impossible, given the very long time scale of the evolutive process, which is in most cases comparable with the time scale of geological changes and anthropogenic interventions; on the other hand, detailed bathymetric data are normally available only for the last century [Blott et al., 2006, e.g.].

Acknowledgments. The authors thank Gianluca Vig-noli for his collaboration in the development of the numerical model. The authors also gratefully acknowledge C. Friedrichs and an anonymous reviewer for their valuable suggestions that allowed for an improvement of the quality of the manuscript.
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References


Dalrymple, R., R. Knight, B. Zaitlin, and G. Middleton (1990), Dynamics and facies model of a macrotidal sandbar complex, Colequid Bay-Salmon River estuary (Bay of Fundy), Sedimentology, 37, 577-612.


Figure 1. Sketch of the estuary and basic notation.

Figure 2. Long-term evolution of the bottom profile of a short convergent estuary ($L_0 = 40 \, km$), (a), and of a long convergent estuary ($L_0 = 240 \, km$), (b), $[L_b = 40 \, km, a_0 = 2 \, m, k_s = 50 \, m^{1/3} \, s^{-1}]$; initial condition: $D_0 = 10 \, m$. 
Figure 3. Temporal evolution of the residual sediment flux $\langle q_s \rangle$, (a), and of the bottom elevation $\eta$, (b), at the mouth ($x = 0$, solid line) and in the middle of the estuary ($x = 0.5 L_0$, dotted line) [$L_b = 40 \text{ km}$, $a_0 = 2 \text{ m}$, $k_s = 50 \text{ m}^{1/3} \text{s}^{-1}$; initial condition: $L_0 = 40 \text{ km}$, $D_0 = 10 \text{ m}$].

Figure 4. Maximum and minimum values along the channel of the velocity $U$, (a), free surface elevation $H$, (b), sediment flux $q_s$, (c), and residual value of the total sediment transport $B \langle q_s \rangle$, (d), at the beginning of the simulation (dashed lines) and at equilibrium (continuous lines) for a short estuary [$L_b = 40 \text{ km}$, $a_0 = 2 \text{ m}$, $k_s = 50 \text{ m}^{1/3} \text{s}^{-1}$; initial condition: $L_0 = 40 \text{ km}$, $D_0 = 10 \text{ m}$].
Figure 5. Velocity $U$, (a), and sediment flux $q_s$, (b), in a tidal cycle at the mouth ($x = 0$) and in the middle of the channel ($x = 0.5 L_0$) at the beginning of the simulation [$L_b = 40 km, a_0 = 2 m, k_s = 50 m^{1/3} s^{-1}$; initial condition: $L_0 = 40 km, D_0 = 10 m$].

Figure 6. Velocity $U$, (a), and sediment flux $q_s$, (b), in a tidal cycle at the mouth ($x = 0$) and in the middle of the channel ($x = 0.5 L_0$) at equilibrium [$L_b = 40 km, a_0 = 2 m, k_s = 50 m^{1/3} s^{-1}$; initial condition: $L_0 = 40 km, D_0 = 10 m$].
Figure 7. Maximum, minimum and residual values along the estuary of velocity $U$, (a), total sediment flux $B_{qs}$, (b), and free surface elevation $H$, (c), at the beginning of the simulation, for different values of the convergence length: $L_b = 15\text{ km ($a_1$, $b_1$)}$, $L_b = 120\text{ km ($a_2$, $b_2$)}$, $L_b \rightarrow \infty ($a_3$, $b_3$) [L_0 = 120\text{ km}, D_0 = 10\text{ m}, a_0 = 2\text{ m}, k_s = 50\text{ m}^{1/3}\text{s}^{-1}].$
Figure 8. The same panel as in Figure 7, at the end of the simulation.

Figure 9. Equilibrium bottom profiles for different values of the convergence length $L_b$, for $L_0 = 40 \, \text{km}$, (a), and $L_0 = 240 \, \text{km}$, (b) [$\omega_0 = 2 \, \text{m}, k_s = 40 \, \text{m}^{1/3} \, \text{s}^{-1}$; initial condition: $D_0 = 10 \, \text{m}$].
Figure 10. Equilibrium bottom profiles for different values of the physical length \( L_0 \), for \( L_b = 40 \text{ km} \), (a), and \( L_b = 80 \text{ km} \), (b) \([a_0 = 2 \text{ m}, k_s = 40 \text{ m}^{1/3} \text{s}^{-1}; \text{initial condition:} \ D_0 = 10 \text{ m}]\).

Figure 11. Equilibrium length of the estuary \( L_e \) as a function of the initial length \( L_0 \), for different values of convergence length \( L_b \) \([a_0 = 2 \text{ m}; \text{initial condition:} \ D_0 = 10 \text{ m}]\).
Figure 12. Equilibrium bottom profiles, (a), and intrinsic equilibrium length $\hat{L}_e$ of the estuary as a function of the convergence length $L_b$, (b), for different values of $k_s$ coefficient [$a_0 = 2m$; (a) $L_b = 80 km$, $L_0 = 240 km$; initial condition: $D_0 = 10 m$].

Figure 13. Equilibrium bottom profiles for different values of the tidal amplitude $a_0$, (a); and, (b), intrinsic equilibrium length $\hat{L}_e$ as a function of the tidal amplitude $a_0$, for different values of $k_s$ coefficient [$L_b = 40 km$; (a) $L_0 = 160 km$, $k_s = 50 m^{1/3} s^{-1}$; initial condition: $D_0 = 10 m$].
Figure 14. Intrinsic equilibrium bottom slope $S$ as a function of the convergence length for different values of $k_s$, (a) [$a_0 = 2$ m]; and for different values of the tidal amplitude $a_0$, (b) [$L_b = 40$ km].

Table 1. Values of the Tidal Amplitude $a_0$, Reference Depth at the Mouth ($D_m$), Length $L_e$, Convergence Length $L_b$, Friction Coefficient $k_s$.

<table>
<thead>
<tr>
<th>Estuary</th>
<th>$a_0$ [m]</th>
<th>$D_m$ [m]</th>
<th>$L_e$ [km]</th>
<th>$L_b$ [km]</th>
<th>$k_s$ [$m^{1/3}$ s$^{-1}$]</th>
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$^a$ Data are taken from Lanzoni and Seminara [1998]; $k_s$ is evaluated from the value reported for $C_h$, using the reference depth $D_m$.

$^b$ Data are modified according to Toffolon et al. [2006].
Figure 15. The estuarine lengths $L_e$ of real estuaries reported in Table 1 are plotted against the convergence lengths $L_b$: a power law interpolating curve is added in order to point out the overall behavior. Results of the present model ($a_0 = 2 \text{ m}$, $k_s = 40 \text{ m}^{1/3} \text{s}^{-1}$, see Figure 12) are reported with a thick line.